Guidance or Resonance Conditions for Strips or Disks Embedded in Homogeneous and Layered Media

LEVENT GUREL, STUDENT MEMBER, IEEE, AND WENG CHO CHEW, SENIOR MEMBER, IEEE

Abstract —We illustrate how the guidance or resonance conditions of strips or disks embedded in layered media can be formulated easily using a new notation we developed. We show that once we know the reflection operator of a reflecting medium, we can find the guidance or resonance conditions of this structure quite easily. We can also find the guidance or resonance conditions when the reflecting medium is interacting with another strip or disk. We illustrate this with the calculations of the guidance of a microstrip line with an infinite ground plane and with a finite ground plane. Our results for the infinite ground plane case agree very well with previous calculations on these problems, while the results for the finite ground plane case are new.

I. Introduction

In this paper, we discuss the formulation of the resonance and guidance conditions for complicated microstrip structures using a notation we developed in the preceding paper [1]. These resonance and guidance conditions allow us to find the resonance frequencies of a structure and the guided mode of a waveguiding structure, respectively. Once we know the reflection operators associated with the structure, the resonance or guidance problem is easily formulated. The derivations of the reflection operators are discussed in the previous paper. These reflection operators together with the new notation have a physical interpretation; hence, we will not lose touch with the underlying physics even for complex problems.

The resonance problem has applications in microstrip resonators and microstrip antennas. We have also applied this formulation to find the resonance frequency of a microstrip disk over an infinite ground plane. In this case, our formulation is similar to that reported in [2] using vector Fourier transform method. However, this formulation can be easily applied to study resonances of more complex structures in microwave integrated circuits.

The guidance problem has applications in microstrip lines and guidance by multiconductor microstrip lines which have applications in high-speed circuitry in computer technology. In this paper, we provide numerical results for a microstrip line over a finite ground plane. In other words, this is the two-conductor microstrip line case. We also discuss the case of a microstrip line over an infinite ground plane. Our results using this method agree with those previously published [3], [4].

The microstrip line waveguide has a long history. Since its appearance, before World War II, there has been continuous work on this waveguide. The early works were quasi-TEM in nature and did not account for the frequency dispersive effects in the waveguides [5]–[11]. The use of this waveguide at higher frequencies called for a frequency dispersive solution. Many methods have been used [12]–[20], but the more popular approach has been the spectral-domain approach (SDA) [21], introduced by Itoh and Mittra [22].

Although a microstrip line with a finite ground plane is encountered in a number of applications, e.g., in antenna feeds, there are surprisingly few publications on the subject in the vast microstrip literature. Some related problems are the microstrip slotline problem reported by Jansen [23] and the work by Itoh on a microstrip line over a slot line using the spectral-domain immittance approach [24].

The geometry of a microstrip transmission line with a finite ground plane is shown in Fig. 1. This problem differs

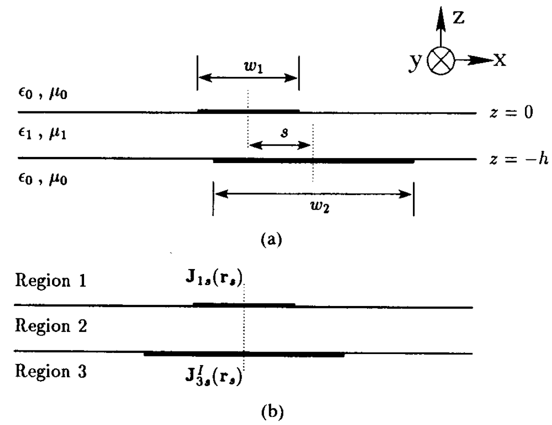

Fig. 1. Microstrip transmission line with a finite ground plane. (a) Shifted strip case. (b) Symmetric case.

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The authors are with the Electromagnetics Laboratory, Department of Electrical and Computer Engineering, University of Illinois, Urbana, IL 61801.

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from the conventional microstrip problem in that the present geometry has a ground plane of width w_2 rather than being infinite. Both the upper strip (of width w_1) and the finite ground plane are infinitely conducting, infinitely thin, and infinitely long in the y direction. An isotropic dielectric slab of thickness h, permittivity ϵ_1, and permeability μ_1 is between the top and the bottom strips. This dielectric slab is of infinite extent. We choose a coordinate system such that the geometry is translationally invariant in the y direction. The goal is to find the propagation constants of all of the modes which are *propagating* in the y direction. The results we present at the end of this paper are for the symmetric case, shown in Fig. 1(b). However, the formulation is given for the more general asymmetric case, shown in Fig. 1(a).

In the microstrip line problem, we shall also discuss how to narrow our search window for the roots of the equation for the guidance condition by requiring the guided microstrip modes to be slower than the surface-wave mode inherent in the dielectric-coated ground plane. By so doing, we also avoid poles in the numerical computation of the integrals involved. We have also studied the effect of the use of subdomain basis functions in this problem and explained the occurrence of the spurious modes.

II. Guidance or Resonance Condition

With the knowledge of the reflection operators defined in [1], we can formulate the guidance or resonance problem quite easily. In [1], we have characterized the scattering of waves from a layered medium with embedded strips by reflection operators. The illuminating source and the subsurface structure can form a resonator in the case of a disk or a waveguide in the case of a strip. In both cases, a solution of Maxwell's equations satisfying all the requisite boundary conditions exists without any other external driving field. Fig. 2 illustrates a strip or a disk at $z = z'$ and an arbitrary geometry which contains other strips and/or disks possibly embedded in a layered medium at $z < 0$. The general expression for the field in region 1 is of the form

$$\boldsymbol{E}_{1s}(\boldsymbol{r}) = \bar{\boldsymbol{F}}(\boldsymbol{r}_s) : \left(e^{i\mathcal{K}_{1z}|z-z'|} + e^{i\mathcal{K}_{1z}z} : \bar{\mathcal{R}} : e^{i\mathcal{K}_{1z}z'}\right) : \bar{\mathcal{G}}_1 : \tilde{\boldsymbol{J}}_s. \tag{1}$$

We assume that the illuminating source is also a strip. The above expression satisfies all boundary conditions except on the illuminating strip. Hence, we require that

$$\boldsymbol{E}_{1s}(\boldsymbol{r}_s, z = z') = \bar{\boldsymbol{F}}(\boldsymbol{r}_s) : \left(\bar{\mathcal{I}} + e^{i\mathcal{K}_{1z}z'} : \bar{\mathcal{R}} : e^{i\mathcal{K}_{1z}z'}\right) : \bar{\mathcal{G}}_1 : \tilde{\boldsymbol{J}}_s = 0, \qquad \boldsymbol{r}_s \in S_1 \tag{2}$$

where S_1 is the surface of the strip in region 1. Expanding

$$\tilde{\boldsymbol{J}}_s(\boldsymbol{k}_s) = \tilde{\boldsymbol{f}}^t(\boldsymbol{k}_s)\cdot\boldsymbol{A} \tag{3}$$

and weighting (2) by $\boldsymbol{f}_t(\boldsymbol{r}_s)$ and integrating over the surface

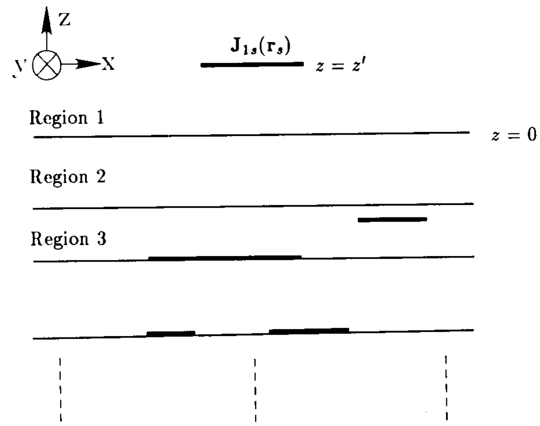

Fig. 2. Arbitrary geometry under a strip or a disk.

of the strip, we have

$$\bar{\Gamma}\cdot\boldsymbol{A} = 0 \tag{4}$$

where

$$\bar{\Gamma} = \tilde{f}_t : \left(\bar{\mathcal{I}} + e^{i\mathcal{K}_{1z}z'} : \bar{\mathcal{R}} : e^{i\mathcal{K}_{1z}z'}\right) : \bar{\mathcal{G}}_1 : \tilde{f}^t. \tag{4a}$$

In order for $\tilde{\boldsymbol{J}}_{1s}(\boldsymbol{r}_s)$ to be nontrivial, we require $\boldsymbol{A}$ to be nontrivial. This implies the existence of a resonant mode or a guided mode. Therefore, the guidance or the resonance condition becomes

$$\det(\bar{\Gamma}) = 0. \tag{5}$$

Another way of stating the guidance or the resonance condition is that the reflection operator associated with the structure is infinite. If we were to define a reflection operator for the total structure above, we would discover that (5) implies that the reflection operator for the structure is infinite.

A. Infinite Ground Plane Case

If the layered medium below the illuminating strip is a substrate backed by an infinite ground plane, then $\bar{\mathcal{R}}$ in (5) is a diagonal operator. In particular, $\bar{\mathcal{R}}$ is an operator representation of the reflection matrix

$$\tilde{\bar{\boldsymbol{R}}} = \begin{bmatrix} -\tilde{R}^{\mathrm{TM}} & 0 \\ 0 & \tilde{R}^{\mathrm{TE}} \end{bmatrix} \tag{6}$$

where [25]

$$\tilde{R}^{\mathrm{TM}} = \frac{R_{01}^{\mathrm{TM}} + e^{i2k_zh}}{1 + R_{01}^{\mathrm{TM}}e^{i2k_zh}} \quad \text{and} \quad \tilde{R}^{\mathrm{TE}} = \frac{R_{01}^{\mathrm{TE}} - e^{i2k_zh}}{1 - R_{01}^{\mathrm{TE}}e^{i2k_zh}}. \tag{7}$$

In (7), R_{01}^{TM} and R_{01}^{TE} are the Fresnel reflection coefficients for the air-dielectric interface and h is the thickness of the dielectric substrate. Then, $\bar{\Gamma}$ to be used in (5) in integral form is

$$\bar{\Gamma} = \left(\int d\boldsymbol{k}_s\, \tilde{\boldsymbol{f}}_t(\boldsymbol{k}_s)\cdot(\bar{\boldsymbol{I}} + \bar{\boldsymbol{R}})\cdot\bar{\boldsymbol{G}}_1(\boldsymbol{k}_s)\cdot\tilde{\boldsymbol{f}}^t(\boldsymbol{k}_s)\right) \tag{8}$$

where $\tilde{\boldsymbol{f}}^t(\boldsymbol{k}_s)$ and $\tilde{\boldsymbol{f}}_t(\boldsymbol{k}_s)$ are column vectors containing the basis functions and testing functions, respectively. If we have N basis functions and N testing functions, (5) is the determinant of a $2N \times 2N$ matrix $\bar{\Gamma}$ whose submatrices (which are 2×2) are given by

$$\bar{\Gamma}_{mn} = \int dk_s \tilde{\bar{J}}^t_{tm}(k_s) \cdot (\bar{I} + \bar{R}) \cdot \bar{G}_1(k_s) \cdot \tilde{\bar{J}}_{sn}(k_s) \quad (9)$$

where $m = 1, \cdots, N$, and $n = 1, \cdots, N$. In the guidance problem, the left-hand side in (5) is a function of k_y, the wavenumber in the direction of propagation of a guided mode. For a resonant problem, (5) is a function of ω, the frequency of the wave.

B. Finite Ground Plane Case

The finite ground plane case, as shown in Fig. 1, consists of an infinite dielectric slab backed by another finite strip or disk, and these are placed right under the illuminating strip. In order to formulate the guidance or the resonance condition in this geometry, we need only to find the appropriate expression for $\bar{\mathscr{R}}$ in (42). Raising the upper strip to $z = z'$ (from $z = 0$) and lowering the finite ground plane to $z = -h-d$ (from $z = -h$), the incident and the scattered fields in region 1 and region 3 can be written as

$$E^O_{1s}(r) = \bar{F}(r_s) : \left(e^{i\mathscr{K}_{1z}|z-z'|} + e^{i\mathscr{K}_{1z}z} : \tilde{\bar{\mathscr{R}}}_{12} : e^{i\mathscr{K}_{1z}z'}\right) : \bar{\mathscr{G}}_1 : \tilde{J}_{1s} \quad (10)$$

$$E^O_{3s}(r) = \bar{F}(r_s) : e^{-i\mathscr{K}_{3z}(z+h)} : \tilde{\bar{\mathscr{T}}}_{13} : e^{i\mathscr{K}_{1z}z'} : \bar{\mathscr{G}}_1 : \tilde{J}_{1s} \quad (11)$$

$$E^S_{3s}(r) = \bar{F}(r_s) : \left(e^{i\mathscr{K}_{1z}|z+h+d|} + e^{-i\mathscr{K}_{3z}(z+h)} : \tilde{\bar{\mathscr{R}}}_{32} : e^{i\mathscr{K}_{3z}d}\right) : \bar{\mathscr{G}}_3 : \tilde{J}^I_{3s} \quad (12)$$

$$E^S_{1s}(r) = \bar{F}(r_s) : e^{i\mathscr{K}_{1z}z} : \tilde{\bar{\mathscr{T}}}_{31} : e^{i\mathscr{K}_{3z}d} : \bar{\mathscr{G}}_3 : \tilde{J}^I_{3s} \quad (13)$$

where $E^O_{is}(r)$ is the original field in region i due to the illuminating strip in the absence of the subsurface strip in region 3, and $E^S_{is}(r)$ is the field in region i due to the induced current on the subsurface strip. In the above, $\tilde{\bar{\mathscr{T}}}_{13}$ is a transmission operator which propagates the field from $z = 0$ in region 1 to $z = -h$. Following the approach given in [1], we expand the induced current density $J^I_{3s}(r_s)$ in terms of $f^t_3(r_s) \cdot A$. We use vector Fourier transforms [26] and require that $E^S_{3s}(r) + E^O_{3s}(r)$ be zero on the lower strip. Letting $z' \to 0$ and $d \to 0$, we can solve for our A and find that

$$A = -\bar{\Gamma}_3^{-1} \cdot \tilde{f}_{3t} : \tilde{\bar{\mathscr{T}}}_{13} : \bar{\mathscr{G}}_1 : \tilde{J}_{1s} \quad (14)$$

where

$$\bar{\Gamma}_3 = \tilde{f}_{3t} : \left(\bar{\mathscr{I}} + \tilde{\bar{\mathscr{R}}}_{32}\right) : \bar{\mathscr{G}}_3 : \tilde{f}^t_3. \quad (15)$$

The reflection operator that we should use in (4a) can be extracted from the expression for the total transverse electric field in region 1 as

$$\bar{\mathscr{R}} = \tilde{\bar{\mathscr{R}}}_{12} - \tilde{\bar{\mathscr{T}}}_{31} : \bar{\mathscr{G}}_3 : \tilde{f}^t_3 \cdot \bar{\Gamma}_3^{-1} \cdot \tilde{f}_{3t} : \tilde{\bar{\mathscr{T}}}_{13}. \quad (16)$$

$\tilde{\bar{\mathscr{T}}}_{13}$ and $\tilde{\bar{\mathscr{T}}}_{31}$ can be derived as shown in the appendix of [1]. Consequently, the reflection operator can be written as

$$\bar{\mathscr{R}} = \tilde{\bar{\mathscr{R}}}_{12} - \bar{\mathscr{T}}_{21} : \left(\bar{\mathscr{I}} - \bar{\mathscr{R}}_{23} : \bar{\mathscr{R}}_{21} : e^{i2\mathscr{K}_{2z}h}\right)^{-1} : e^{i\mathscr{K}_{2z}h} : \bar{\mathscr{T}}_{32} : \bar{\mathscr{G}}_3 : \tilde{f}^t_3 \cdot \bar{\Gamma}_3^{-1} \cdot \tilde{f}_{3t} : \bar{\mathscr{T}}_{23} : e^{i\mathscr{K}_{2z}h} : \left(\bar{\mathscr{I}} - \bar{\mathscr{R}}_{21} : \bar{\mathscr{R}}_{23} : e^{i2\mathscr{K}_{2z}h}\right)^{-1} : \bar{\mathscr{T}}_{12}. \quad (17)$$

In the above derivation, $\tilde{\bar{\mathscr{R}}}_{12}$ and $\tilde{\bar{\mathscr{R}}}_{32}$ are generalized reflection operators due to the slab alone. Substituting (17) into (5), we have

$$\det\Big\{ \tilde{f}_{1t} : \left(\bar{\mathscr{I}} + \tilde{\bar{\mathscr{R}}}_{12}\right) : \bar{\mathscr{G}}_1 : \tilde{f}^t_1 - \tilde{f}_{1t} : \bar{\mathscr{T}}_{21} : \left(\bar{\mathscr{I}} - \bar{\mathscr{R}}_{23} : \bar{\mathscr{R}}_{21} : e^{i2\mathscr{K}_{2z}h}\right)^{-1} : e^{i\mathscr{K}_{2z}h} : \bar{\mathscr{T}}_{32} : \bar{\mathscr{G}}_3 : \tilde{f}^t_3 \cdot \bar{\Gamma}_3^{-1} \cdot \tilde{f}_{3t} : \bar{\mathscr{T}}_{23} : e^{i\mathscr{K}_{2z}h} : \left(\bar{\mathscr{I}} - \bar{\mathscr{R}}_{21} : \bar{\mathscr{R}}_{23} : e^{i2\mathscr{K}_{2z}h}\right)^{-1} : \bar{\mathscr{T}}_{12} : \bar{\mathscr{G}}_1 : \tilde{f}^t_1 \Big\} = 0. \quad (18)$$

If we assume that medium 1 and medium 3 are identical, then the above is of the form

$$\det\left\{ \bar{\Gamma}_1 - \bar{\Gamma}_{13} \cdot \bar{\Gamma}_3^{-1} \cdot \bar{\Gamma}^t_{13} \right\} = 0 \quad (19)$$

where

$$\bar{\Gamma}_1 = \tilde{f}_{1t} : \left(\bar{\mathscr{I}} + \tilde{\bar{\mathscr{R}}}_{12}\right) : \bar{\mathscr{G}}_1 : \tilde{f}^t_1 \quad (20a)$$

and

$$\bar{\Gamma}_{13} = \tilde{f}_{1t} : \bar{\mathscr{T}}_{21} : \left(\bar{\mathscr{I}} - \bar{\mathscr{R}}^2_{21} : e^{i2\mathscr{K}_{2z}h}\right)^{-1} : e^{i\mathscr{K}_{2z}h} : \bar{\mathscr{T}}_{12} : \bar{\mathscr{G}}_1 : \tilde{f}^t_3. \quad (20b)$$

Hence, all the $\bar{\Gamma}$ matrices in (20) are computable. In the above, the $\bar{\Gamma}_i$ matrix is responsible for the self-interaction of the current on the strip in region i, whereas $\bar{\Gamma}_{ij}$ is a cross-interaction matrix responsible for the interaction of the current on the strip in region i with the current on the strip in region j. The $\bar{\Gamma}_i$ matrices can be made symmetrical by a proper choice of the expansion functions and the testing functions. The $\bar{\Gamma}_{ij}$ matrix is asymmetrical.

III. Computational Notes

We have formulated the strip or the disk problem embedded in a layered medium. The disk problem is useful in microstrip resonators. In this case, we retain all the previous definitions. The strip problem is useful in waveguides. For waveguiding structures, we can assume that the field has $e^{ik_y y}$ dependence, where y is the direction of propagation. In the strip case, the integral $\int dk_s$ becomes $\int dk_x$ and $1/4\pi^2$ becomes $1/2\pi$.

In the disk case, the testing function $\bar{J}_{tm}(r_s)$ is defined to be

$$\bar{J}_{tm}(r_s) = \begin{bmatrix} J_{xm}(r_s) & 0 \\ 0 & J_{ym}(r_s) \end{bmatrix}. \quad (21)$$

In this case, the matrix $\bar{\Gamma}$ is symmetric. $\bar{\Gamma}$ is of the general form

$$\bar{\Gamma} = \tilde{f}_t(\boldsymbol{k}_s) : \bar{\bar{\mathscr{D}}} : \tilde{f}^t(\boldsymbol{k}_s) \tag{22}$$

where $\bar{\bar{\mathscr{D}}}$ is a diagonal operator. For the strip case, in order to obtain a symmetric $\bar{\Gamma}$, we can define

$$\bar{J}_{tm}(\boldsymbol{r}_s) = \begin{bmatrix} J_{xm}(\boldsymbol{r}_s) & 0 \\ 0 & -J_{ym}(\boldsymbol{r}_s) \end{bmatrix}. \tag{23}$$

Symmetric $\bar{\Gamma}$ matrices save computation time in the filling of the matrices.

A. Microstrip Transmission Line with an Infinite Ground Plane

For the microstrip transmission line with an infinite ground plane, the matrix $\bar{\Gamma}$ is given by

$$\bar{\Gamma} = \int_{-\infty}^{\infty} dk_x \tilde{f}(k_x) \cdot (\bar{I} + \bar{R}) \cdot \bar{G}_1(k_x) \cdot \tilde{f}^t(k_x). \tag{24}$$

In solving for k_y in the guidance problem for a microstrip line with an infinite ground plane, we need to find k_y that makes the determinant of $\bar{\Gamma}$ go to zero. A natural window of search for k_y is

$$k = \omega\sqrt{\mu_0\epsilon_0} < k_y < \omega\sqrt{\mu_1\epsilon_1} = k_1. \tag{25}$$

However, for some values of k_y in this window together with some values of k_x, the dyadic Green's function, $(\bar{I} + \bar{R}) \cdot \bar{G}_1(k_x)$, becomes singular, thus rendering the integral of (24) difficult to compute if we integrate on the real axis. This happens if either

$$1 + R_{01}^{\text{TM}} e^{i2k_z h} = 0 \tag{26a}$$

or

$$1 - R_{01}^{\text{TE}} e^{i2k_z h} = 0. \tag{26b}$$

Equations (26a) and (26b) are recognized as the guidance conditions for TM and TE modes, respectively, propagating in the dielectric-coated ground plane. These modes are the surface-wave modes in the dielectric-coated ground plane. They can propagate in any direction in the xy plane. Since we are looking at a guided mode on the microstrip line, the guided mode should not leak energy into the surface-wave modes. Otherwise, the surface-wave modes will carry energy away from the microstrip line. When that happens, the microstrip line modes are cut off. In order for the microstrip mode to be trapped in the microstrip line entirely, we require the surface-wave mode to be evanescent away from the microstrip line. In other words, we require the microstrip line modes to be slower than the surface-wave mode. If the surface-wave mode has a wavenumber transverse to z which is k_p, then we require that k_y for the microstrip mode be larger than k_p.

Furthermore, when $k_y > k_p$, the poles of the $(\bar{I} + \bar{R}) \cdot \bar{G}_1(k_x)$ are all on the imaginary k_x axis on the complex k_x plane. This ensures the convergence of the numerical computation of the integral in (24) on the real k_x axis. Therefore, we can shrink the window of search to

$$k_p < k_y < k_1. \tag{27}$$

In this manner, we can integrate on the real k_x axis without the fear of encountering a pole. Finally, it is sufficient to search for k_p for the fundamental mode only, i.e., the TM_0 mode, since the k_p for the fundamental mode is always larger than that of the higher order modes. Thus, k_p is the largest root of (26a) with k_x set to zero.

Explicit forms of the basis functions will be given in the next subsection. The results obtained using these basis functions are compared to those of Jansen [3] and Kobayashi and Ando [4] and are found to be either in excellent agreement or even more precise in some cases.

We have also used subdomain basis functions such as the triangle or *chapeau* functions. These functions form a piecewise-linear approximation of the currents on the strip. In this case, we discovered the occurrence of spurious modes as had been previously reported [3]. It was found that the basis functions had to have continuous derivatives before these spurious modes disappear. This may be due to the fact that the use of basis functions with discontinuous derivatives gives rise to fictitious charges. These fictitious charges may be responsible for the occurrence of these spurious modes.

Our formulation is novel to the microstrip problem. This method is more general and can encompass a larger class of problems in the microwave integrated circuits area. The formulation is in terms of Green's functions and reflection operators which have physical meanings closer to those for the field theory rather than circuit theory. With the appropriate choice of testing functions, the final matrix $\bar{\Gamma}$ is symmetric. Furthermore, since the basis functions are either odd or even symmetric and the Green's function is even symmetric with respect to k_x, half of the elements of the matrix $\bar{\Gamma}$ are set equal to zero *a priori*. These two symmetry properties result in the computation of only 25 percent of the matrix elements as the matrix size goes to infinity.

B. Microstrip Transmission Line with a Finite Ground Plane

In the finite ground plane case, $\bar{\Gamma}_1$, $\bar{\Gamma}_{13}$, and $\bar{\Gamma}_3$ are given by

$$\bar{\Gamma}_1 = [\bar{\Gamma}_{1mn}]_{2M \times 2M} \tag{28}$$

where

$$\bar{\Gamma}_{1mn} = \int_{-\infty}^{\infty} dk_x \tilde{\bar{J}}_{1tm}^t \cdot (\bar{I} + \tilde{\bar{R}}_{12}) \cdot \bar{G}_1 \cdot \tilde{\bar{J}}_{1sn} \tag{29}$$

$$\bar{\Gamma}_{13} = [\bar{\Gamma}_{13mn}]_{2M \times 2N} \tag{30}$$

where

$$\bar{\Gamma}_{13mn} = \int_{-\infty}^{\infty} dk_x \tilde{\bar{J}}_{1tm}^t \cdot \bar{T}_{21} \cdot \left(\bar{I} - e^{i2k_{2z}h} \bar{R}_{21}^2\right)^{-1} \cdot e^{ik_{2z}h} \bar{T}_{12} \cdot \bar{G}_1 \cdot \tilde{\bar{J}}_{3sn} \tag{31}$$

and

$$\bar{\Gamma}_3 = [\bar{\Gamma}_{3mn}]_{2N \times 2N} \tag{32}$$

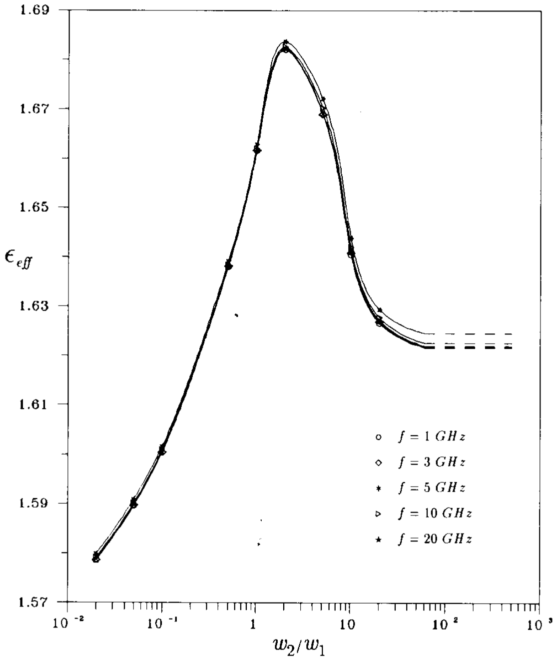

Fig. 3. Effective dielectric constant of the fundamental odd mode when $\epsilon_r = 2.0$ and $h = 1.6w_1$.

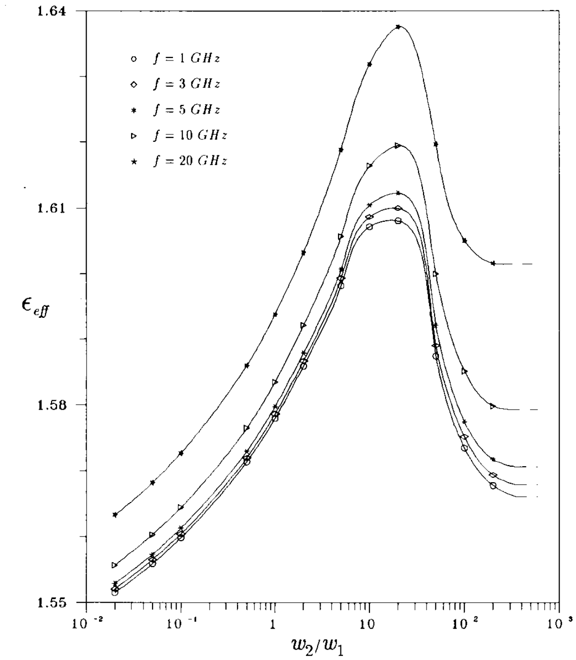

Fig. 4. Effective dielectric constant of the fundamental odd mode when $\epsilon_r = 2.0$ and $h = 12.8w_1$.

where

$$\bar{\Gamma}_{3mn} = \int_{-\infty}^{\infty} dk_x \tilde{\bar{J}}^t_{3tm} \cdot \left(\bar{I} + \tilde{\bar{R}}_{32}\right) \cdot \bar{G}_1 \cdot \tilde{\bar{J}}_{3sn} \tag{33}$$

assuming that M basis function matrices are used to expand the surface current density on the upper strip and N basis function matrices are used for the lower strip.

Due to the formulation of the problem, those values of k_y that satisfy the guidance condition of (19) are the wavenumbers of the guided modes in the y direction.

The window of search for the k_y values is again determined in such a way that convergence of the numerical integrations of (29), (31), and (33) is ensured. We see that the integrands become infinite when

$$\bar{I} - e^{i2k_{2z}h}\bar{R}^2_{21} = 0 \tag{34}$$

which is precisely the guidance condition in the dielectric slab. Slab modes are guided in all directions on the xy plane. Therefore, they carry power away from the strips and cause the microstrip modes to be cut off. In order to avoid this situation, the range of k_y that causes the slab modes to be evanescent in all directions except the y direction should be found. Following the discussion of the infinite ground plane case, (34) is solved for both the TM and TE components to obtain a number of k_p^{TM}'s and k_p^{TE}'s, the wavenumbers transverse to z of the guided TM and TE modes in the dielectric slab. If we call the largest of these k_p^{TM}'s and k_p^{TE}'s to be k_p, then the window of search is shrunk to

$$k_p < k_y < k_1 = \omega\sqrt{\mu_1\epsilon_1}\,. \tag{35}$$

As long as (35) is satisfied, the poles of integrands are not on the real k_x axis and convergence of integrals is ensured when we integrate on the real k_x axis.

The set of basis functions we use is the Chebyshev polynomials modified according to the edge conditions as used by Poh *et al.* [27]. Since the lower strip is allowed to shift by an amount s as shown in Fig. 1(a), basis functions for the lower strip are chosen as the shifted version of the modified Chebyshev polynomials. Therefore, $J_{3yn}(x)$ is given by

$$J_{3yn}(x) = \begin{cases} T_n\left(\dfrac{2(x-s)}{w_2}\right)\left\{1 - \left(\dfrac{2(x-s)}{w_2}\right)^2\right\}^{-1/2} & \text{if } |x-s| < \dfrac{w_2}{2} \\ 0 & \text{if } |x-s| > \dfrac{w_2}{2} \end{cases} \tag{36}$$

where w_2 is the width of the bottom strip. Consequently, $\tilde{J}_{3yn}(k_x)$, $J_{3xn}(x)$, and $\tilde{J}_{3xn}(k_x)$ are found to be

$$\tilde{J}_{3yn}(k_x) = \frac{w_2}{2}(-i)^n \pi J_n\left(\frac{w_2 k_x}{2}\right)e^{-ik_x s} \tag{37}$$

$$J_{3xn}(x) = \int_{-\infty}^{x} J_{2yn}(\xi)\,d\xi \tag{38}$$

$$\tilde{J}_{3xn}(k_x) = \frac{w_2}{2}(-1)^{n+1}\frac{\pi}{k_x}J_n\left(\frac{w_2 k_x}{2}\right)e^{-ik_x s}. \tag{39}$$

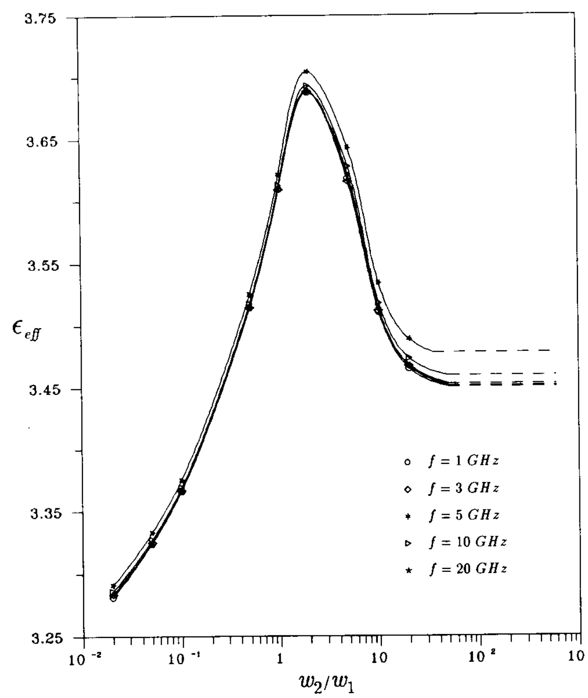

Fig. 5. Effective dielectric constant of the fundamental odd mode when $\epsilon_r = 5.0$ and $h = 1.6w_1$.

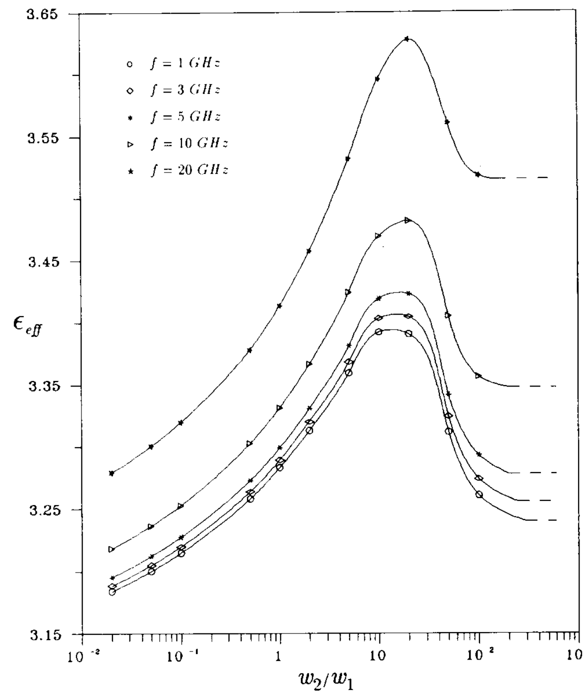

Fig. 6. Effective dielectric constant of the fundamental odd mode when $\epsilon_r = 5.0$ and $h = 12.8w_1$.

The basis functions of the upper strip in both the space domain and the spectral domain are also given by (36) to (39) provided that w_2 is replaced by w_1 and s is set to zero. For the case of symmetric strips as shown in Fig. 1(b), s is also set to zero for the basis functions of the lower strip.

As a final note, the Green's function for an electromagnetic field is highly singular. This translates to the case where the Green's function decays slowly for high spectral frequencies in the spectral domain. In order to expedite the convergence of the spectral-domain integrals, we can subtract out the slowly convergent part of the integral that is associated with the singularity of the Green's function. Since this part is independent of k_y, it needs to be done only once, and used later when we are changing the values of k_y to look for the roots of the determinant in (5).

Some of the results obtained using the formulation and basis functions of this section are presented in the next section.

IV. Results and Conclusions

A microstrip transmission line with a finite ground plane has both even and odd modes, contrary to the infinite ground plane case where only the odd modes exist. We define an odd mode to be when the current on the top conductor has opposite polarity to the current on the bottom conductor. An even mode is when the current on the top conductor has the same polarity as the current on the bottom conductor. An infinite ground plane problem is equivalent to a finite ground plane problem in which the upper and lower strips have equal widths [28]. We can show that the equivalent finite ground plane geometry has more guided modes than the corresponding infinite ground plane geometry. We can also prove that the extraneous finite ground plane modes are even modes by postulating another equivalent problem as shown in [28]. When the width of the finite ground plane is equal to the width of the upper strip, there are two corresponding equivalent problems: one, a perfect electric conductor inserted midway between the top and the bottom strips, the other, a perfect magnetic conductor inserted midway between the top and the bottom strips.

The odd mode of the finite ground plane problem has a higher effective dielectric constant than that of the even mode, because more field is trapped within the dielectric slab for the odd mode. The effective dielectric constant in this context is defined to be

$$\epsilon_{\text{eff}} = \left(\frac{k_y}{k}\right)^2 \tag{40}$$

where k_y is the propagation constant of the mode and k is the free-space wavenumber. We consider the odd mode as the fundamental mode, and most of the results we present in this section are calculated for the odd mode.

In Figs. 3 to 11, we show plots of ϵ_{eff} for the odd and even modes of the finite ground plane problem when $s = 0$. In all of these plots, the width of the upper strip (w_1) is set equal to 0.1 mm and the data points are obtained by changing the relative dielectric constant (ϵ_r) of the sub-

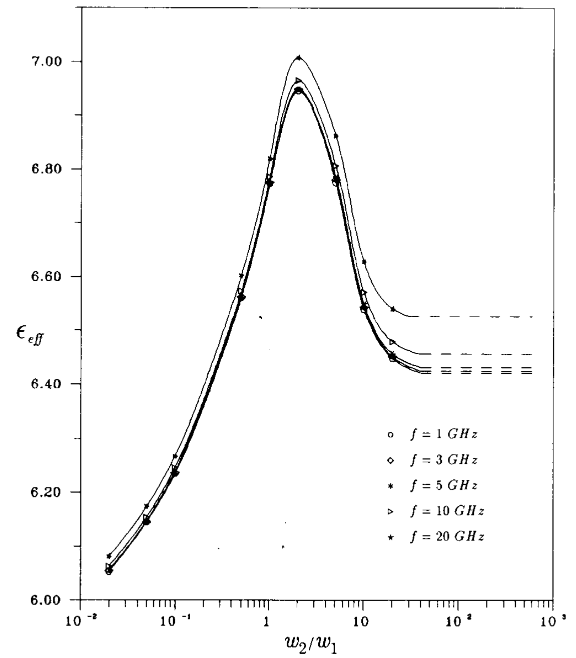

Fig. 7. Effective dielectric constant of the fundamental odd mode when $\epsilon_r = 9.9$ and $h = 1.6w_1$.

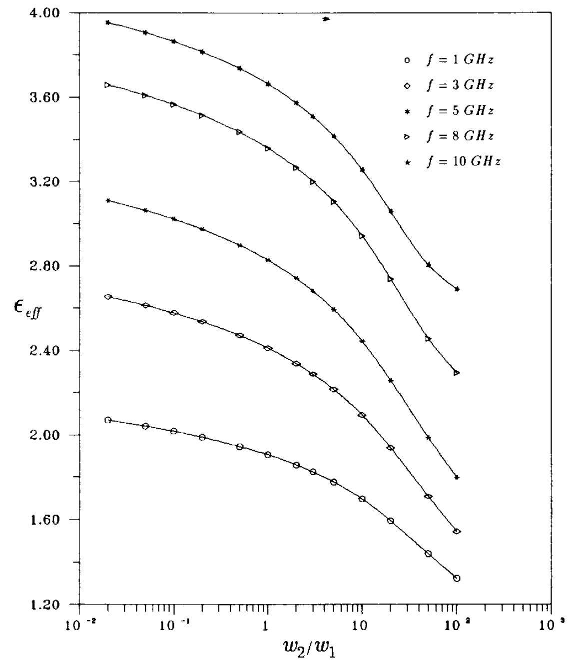

Fig. 9. Effective dielectric constant of the even mode when $\epsilon_r = 9.9$ and $h = 12.8w_1$.

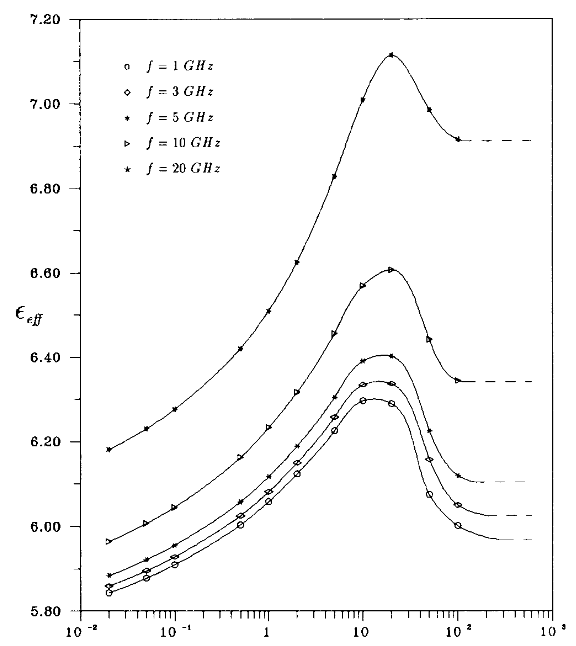

Fig. 8. Effective dielectric constant of the fundamental odd mode when $\epsilon_r = 9.9$ and $h = 12.8w_1$.

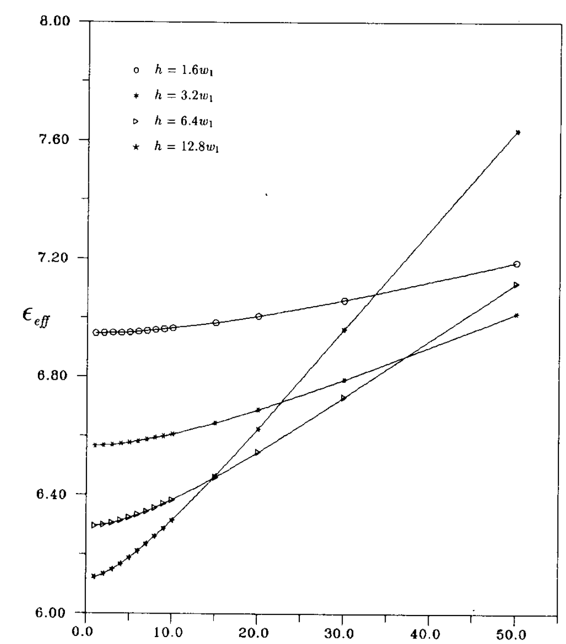

Fig. 10. Effective dielectric constant of the fundamental odd mode when $\epsilon_r = 9.9$ and $w_2/w_1 = 2$.

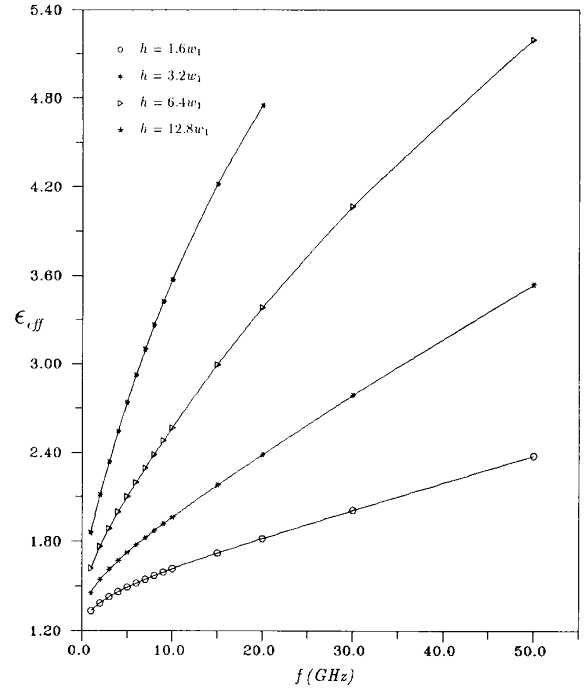

Fig. 11. Effective dielectric constant of the even mode when $\epsilon_r = 9.9$ and $w_2/w_1 = 2$.

strate, the thickness (h) of the dielectric substrate, the ratio of the widths of the strips (w_2/w_1), and the frequency (f) as parameters. Data points are shown by markers and they are interpolated using a spline technique.

In Figs. 3 to 8, we plot ϵ_{eff} versus w_2/w_1 of the fundamental odd mode for the relative dielectric constants of 2.0, 5.0, and 9.9, and for substrate thickness of $h = 1.6w_1$ and $h = 12.8w_1$. We see that the frequency dispersion becomes appreciable when the thickness of the dielectric substrate increases. Although not shown here, the dispersion effect is negligible for $h = 0.2w_1$. The plots are given as a function of w_2/w_1 so that the asymptotic behavior as $w_2 \to \infty$ can be seen. The dashed lines represent the effective dielectric constant of the corresponding infinite ground plane problem ($w_2 \to \infty$). As the ratio w_2/w_1 increases, ϵ_{eff} approaches that of the infinite ground plane case. However, contrary to our expectation, ϵ_{eff} peaks before converging to its value corresponding to that for the infinite ground plane case. This peak can be as high as $0.06\epsilon_r$ above the asymptotic value of ϵ_{eff} as $w_2/w_1 \to \infty$, which may not be ignored in some situations. In Figs. 3 to 8, we show what value of the ratio w_2/w_1 one should have in order to reach the performance of the infinite ground plane case. Roughly, this ratio is around 50 when $h = 1.6w_1$ and can reach 300 when $h = 12.8w_1$. However, this ratio is below 10 when $h = 0.2w_1$ (not shown here).

In Fig. 9, we show the effective dielectric constants of even modes for $\epsilon_r = 9.9$, $h = 12.8w_1$, and $f = 1, 3, 5, 10, 20$ GHz as a function of w_2/w_1. Figs. 10 and 11 are the plots of odd and even modes, respectively, for $\epsilon_r = 9.9$, $w_2/w_1 = 2$, and $h/w_1 = 1.6, 3.2, 6.4, 12.8$ as a function of frequency. In Figs. 8 to 11, we depict the fact that effective dielectric constants of the even modes are much lower than those of the corresponding odd modes. Again this occurs because more field is trapped within the dielectric slab for the odd mode.

This concludes the discussion of guidance problem in the microstrip transmission line with a finite ground plane. Many other problems with geometries involving strips and stratified media can be solved following the methodology presented in this paper.

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Levent Gurel (S'87) was born in Izmir, Turkey, on May 12, 1964. He received the B.Sc. degree from the Middle East Technical University, Ankara, Turkey, in 1986 and the M.S. degree from the University of Illinois, Urbana, in 1988, both in electrical engineering. Currently, he is a Ph.D. student and a research assistant at the University of Illinois.

Weng Cho Chew (S'79–M'80–SM'86) was born on June 9, 1953, in Malaysia. He received the B.S. degree in 1976, both the M.S. and Engineer's degrees in 1978, and the Ph.D. degree in 1980, all in electrical engineering, from the Massachusetts Institute of Technology, Cambridge.

His research interest has been in the area of wave propagation and interaction with heterogeneous media for geophysical subsurface sensing, microwave and millimeter-wave integrated circuits, and microstrip antenna applications. He has also studied electrochemical and dielectric properties of rocks. From 1981 to 1985, he was with Schlumberger-Doll Research in Ridgefield, CT. While there, eh was a program leader and a department manager. Currently, he is an Associate Professor with the Department of Electrical and Computer Engineering, University of Illinois at Urbana–Champaign.

Dr. Chew is a member of Eta Kappa Nu, Tau Beta Pi, and URSI and an active member with the Society of Exploration Geophysics. He is an NSF Presidential Young Investigator for 1986. He is an Associate Editor and AdCom member with the IEEE Geoscience and Remote Sensing Society, and served as a Guest Editor with *Radio Science*.