Reflection and Transmission Operators for Strips or Disks Embedded in Homogeneous and Layered Media

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Abstract —In this paper, we introduce a new notation to simplify the solution of scattering by strips or disks. We make use of vector Fourier transforms [1] and introduce a double dot product for inner products in an uncountably infinite dimensional linear vector space. We characterize the scattering by a strip or a disk with a reflection operator and a transmission operator that relate the continuum of scattered waves to a continuum of incident waves. After the reflection operator for a single strip or disk is derived, we show how the reflection operator for a strip or disk in the presence of another reflecting medium, e.g., a layered medium, can be derived. The scattering by N strips or disks in a homogeneous medium is also discussed. Next, we derive the reflection operator for an embedded strip or disk in a layered medium. The method can be generalized to N strips or disks embedded in a layered medium and to a slot or an aperture. These operators have applications in a number of scattering, guidance, and resonance problems. In the paper that follows this one, we shall show how such concepts can be used to formulate the guidance and resonance problems involving N strips or disks whose reflection operator is known.

I. Introduction

EVEN THOUGH microstrip transmission lines have been around since before World War II, the research on the analysis of microwave integrated circuits (MIC's) is still in progress. A review of this progress is available in [2]. This is partly due to the growing importance of monolithic microwave integrated circuits (MMIC's) and high-speed circuitry in computer technology. A better capability in the analysis of microwave integrated circuits is needed to advance the technology in this field.

In the beginning, the analyses of microwave integrated circuits were mainly involved with microstrip lines. Analytic solutions were important at the beginning due to the skimpy computation power then available [3]–[5]. However, the computer technology has called for numerical methods in which the solutions can be obtained more precisely. There have been extensive publications on this topic and we have listed some of these publications in [6]–[22]. This list is by no means complete. Many of the analyses involve the solution of integral equations in the Fourier transform domain [10]–[22] rather than the real space domain, because microwave integrated circuits are built on top of a dielectric substrate. The Green's function for such a class of problems exists in a simple form in the Fourier transform domain or the spectral domain. A related idea is found in the literature on mixed boundary-value problems and dual integral equations [23]. Itoh and Mittra were the first to bring these concepts to the microwave community [10].

The need to push for higher speed circuitry in computer technology, the growing complexity of MMIC's, and the recent interest in microstrip antennas and arrays all contribute to the growing complexity of the geometry involved in microwave integrated circuits. However, these problems need to be solved despite their complexities.

In this paper, we review the past work done in this area and introduce a new formulation for this class of problems. We will introduce a new notation which will make the formulation of such a class of problems simpler. With the introduction of the new notation, the underlying physics of the scattering processes is not lost, but is clarified. We will show how to use the new notation, together with vector Fourier transforms [1], to derive the reflection and transmission operators of a class of problems ranging from a single strip or disk to N embedded strips or disks in a layered medium. The reflection and transmission operators relate the continuum of scattered waves to a continuum of incident waves. Since the new notation elucidates the physics better, we are able to give our operators physical interpretations. Furthermore, all the operators we have derived are computable.

Since we are introducing a number of new concepts in this work, we will divide the work into two parts. The first part is mainly concerned with the description and interpretation of the new notation and the use of vector Fourier transforms in the derivation of the reflection and transmission operators. These operators have applications in some scattering problems, for example, in calculating the radar scattering cross section of frequency selective surfaces, and in some resonance and guidance problems. In the second part of the work, which will be reported in the next paper

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(pp. 1498–1506, this issue), we shall illustrate the use of such concepts to calculate certain guidance and resonance problems related to microwave integrated circuits.

II. Basic Conventions

A. Operators and Double Dot Product

We will introduce some new notations here which will help simplify the formulation of this class of problems. With these notations, complicated and long expressions can be written compactly. These notations also elucidate the physics of the problem better. Hence, they can help in the solutions of complicated scattering and guidance problems.

Consider any field $\phi(\boldsymbol{r})$, which is a solution of the wave equation and which is due to a source at the origin. It can be written in a homogeneous medium via the use of Fourier transforms as

$$\phi(\boldsymbol{r}) = \int d\boldsymbol{k}_s e^{i\boldsymbol{k}_s\cdot\boldsymbol{r}_s}\tilde{\psi}(\boldsymbol{k}_s)e^{ik_z|z|} \quad (1)$$

where $\boldsymbol{k}_s = \hat{x}k_x + \hat{y}k_y$, $\boldsymbol{r}_s = \hat{x}x + \hat{y}y$, $k_z = \sqrt{k^2 - k_s^2}$, and $\int d\boldsymbol{k}_s = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dk_x\, dk_y$. In the above, the special form of the integrand follows from enforcing the field to be a solution of the wave equation and requiring the field to be outgoing at $z = \pm\infty$ (with $\exp(-i\omega t)$ time convention). The integral can be thought of as an integral summation of plane waves (including the inhomogeneous plane waves as well). When an inhomogeneity, which is translationally invariant in the x and y directions, is introduced at $z = -d$, $\phi(\boldsymbol{r})$ becomes

$$\phi(\boldsymbol{r}) = \int d\boldsymbol{k}_s e^{i\boldsymbol{k}_s\cdot\boldsymbol{r}_s}\tilde{\psi}(\boldsymbol{k}_s)\left(e^{ik_z|z|} + Re^{i2k_z d}e^{ik_z z}\right) \quad \text{for } z > -d. \quad (2)$$

In the above, R is the appropriate reflection coefficient, which can be either a Fresnel reflection coefficient, as in the case of a half-space interface, or a generalized reflection coefficient, as in the case of a layered medium. In the integrand, we see that a plane wave incident with a transverse wave vector $\boldsymbol{k}_s$ will always be reflected with the same transverse wave vector $\boldsymbol{k}_s$.

However, if the interface is not translationally invariant in the x and y directions, then the reflected field expression is not related to the incident field expression just by using a simple reflection coefficient. This is the case, for instance, in the problem of scattering from a finite-width strip. An incident wave with a transverse wavenumber $\boldsymbol{k}_s$ gives rise to a spectrum of reflected and transmitted waves with different transverse wavenumbers $\boldsymbol{k}_s'$. In this case, we can define a reflection operator that properly describes the scattering process. Hence, we can express the field as

$$\phi(\boldsymbol{r}) = \int d\boldsymbol{k}_s e^{i\boldsymbol{k}_s\cdot\boldsymbol{r}_s}\left(\tilde{\psi}(\boldsymbol{k}_s)e^{ik_z|z|} + e^{ik_z(z+d)}\int d\boldsymbol{k}_s' R(\boldsymbol{k}_s,\boldsymbol{k}_s')e^{ik_z' d}\tilde{\psi}(\boldsymbol{k}_s')\right) \quad \text{for } z > -d \quad (3)$$

if the interface is at $z = -d$ and $k_z' = \sqrt{k^2 - k_s'^2}$. Equation (3) can be interpreted to mean that each reflected wave with a transverse wavenumber $\boldsymbol{k}_s$ is a consequence of a spectrum of incident waves with different transverse wavenumbers $\boldsymbol{k}_s'$; hence, we have an integral over $\boldsymbol{k}_s'$.

The reflection operator of (3) involves an integral with respect to $\boldsymbol{k}_s'$. It would be very desirable to cast this operator into a simpler notation since we will be using it extensively. We can think of an integral as the limiting case of an infinite sum. Such an infinite sum S may be given by

$$S = \sum_n \Delta\boldsymbol{k}_s R(m\Delta\boldsymbol{k}_s, n\Delta\boldsymbol{k}_s)e^{ik_{zn}d} = \sum_n R_{mn}e^{ik_{zn}d} \quad (4)$$

where $k_{zn} = \sqrt{k^2 - (n\Delta\boldsymbol{k}_s)^2}$ and R_{mn} is defined as $\Delta\boldsymbol{k}_s R(m\Delta\boldsymbol{k}_s, n\Delta\boldsymbol{k}_s)$. The above can be written as a matrix inner product (dot product), that is,

$$S = \overline{\boldsymbol{R}}\cdot e^{i\boldsymbol{K}_z d} \quad (5)$$

where

$$\overline{\boldsymbol{R}} = \begin{bmatrix} \ddots & \vdots & \\ \cdots & R_{mn} & \cdots \\ & \vdots & \ddots \end{bmatrix} \quad \text{and} \quad e^{i\boldsymbol{K}_z d} = \begin{bmatrix} \vdots \\ e^{ik_{zn}d} \\ \vdots \end{bmatrix}. \quad (6)$$

The integral I given by

$$I = \int d\boldsymbol{k}_s' R(\boldsymbol{k}_s,\boldsymbol{k}_s')e^{ik_z' d} \quad (7)$$

as it appears in (3) is a limiting case of the summation S when the index n now changes continuously or is a continuum. Therefore, we define an operator $\mathscr{R}$ which is an extension of a matrix operator with infinitely many rows and infinitely many columns and which is continuously indexed; i.e., $\mathscr{R}$ is the version of $\overline{\boldsymbol{R}}$ where m and n are continuum quantities. Also, we define an infinitely long vector $e^{ik_z z}$ which is the continuum version of $e^{i\boldsymbol{K}_z d}$. Finally, we define a double dot product as an alternative to an ordinary dot product to multiply continuously indexed matrices and vectors. Then, the integral I can be written compactly as

$$I = \mathscr{R}:e^{ik_z z}. \quad (8)$$

An integration over the $\boldsymbol{k}_s$ variable is implied in the double dot product. An operator in the above definition is a representation of a function with two arguments, e.g., $\mathscr{R}(\boldsymbol{k}_s,\boldsymbol{k}_s')$, that maps a single argument function defined over $\boldsymbol{k}_s'$ to another single argument function defined over $\boldsymbol{k}_s$. We shall denote operators with "calligraphic" characters in the rest of the paper. From the above definition, an identity operator is just the Dirac delta function $\delta(\boldsymbol{k}_s - \boldsymbol{k}_s')$.

Below are some examples related to the use of the double dot product. The first is

$$\int d\boldsymbol{k}_s A(\boldsymbol{k}_s) B(\boldsymbol{k}_s) C(\boldsymbol{k}_s) = \int d\boldsymbol{k}_s A(\boldsymbol{k}_s) \int d\boldsymbol{k}_s' \delta(\boldsymbol{k}_s - \boldsymbol{k}_s') B(\boldsymbol{k}_s') C(\boldsymbol{k}_s') = A : \mathscr{B} : C \tag{9a}$$

where $\mathscr{B}$ is a diagonal operator which is a representation of the two argument function $\delta(\boldsymbol{k}_s - \boldsymbol{k}_s')B(\boldsymbol{k}_s')$. Hence, the diagonal operator is related to the single argument function $B(\boldsymbol{k}_s')$, and we shall call $B(\boldsymbol{k}_s')$ the diagonal element of the diagonal operator $\mathscr{B}$. In the above, two integrals are implicit in the two double dot products. It is also the continuum analogue of $\sum_n A_n B_n C_n$ which can be written as $\boldsymbol{A}^t \cdot \overline{\boldsymbol{B}} \cdot \boldsymbol{C}$ where $\boldsymbol{A}$ and $\boldsymbol{C}$ are column vectors containing A_n and C_n, respectively, and $\overline{\boldsymbol{B}}$ is a diagonal matrix containing B_n on the diagonal. The next example is

$$\int d\boldsymbol{k}_s \boldsymbol{A}^t(\boldsymbol{k}_s) \cdot \overline{\boldsymbol{B}}(\boldsymbol{k}_s) \cdot \boldsymbol{C}(\boldsymbol{k}_s) = \int d\boldsymbol{k}_s \boldsymbol{A}^t(\boldsymbol{k}_s) \cdot \int d\boldsymbol{k}_s' \delta(\boldsymbol{k}_s - \boldsymbol{k}_s') \overline{\boldsymbol{B}}(\boldsymbol{k}_s') \cdot \boldsymbol{C}(\boldsymbol{k}_s') = \boldsymbol{A}^t : \overline{\mathscr{B}} : \boldsymbol{C} \tag{9b}$$

where $\overline{\mathscr{B}}$ is a block diagonal operator. The third example is

$$\int d\boldsymbol{k}_s R(\boldsymbol{k}_s, \boldsymbol{k}_s') S(\boldsymbol{k}_s) = \mathscr{R} : S \tag{9c}$$

where $\mathscr{R}$ is a nondiagonal (full) operator. The last example is

$$\int d\boldsymbol{k}_s \overline{\boldsymbol{F}} \cdot e^{ik_z z} \overline{\boldsymbol{G}}(\boldsymbol{k}_s) \cdot \int d\boldsymbol{k}_s' \overline{\boldsymbol{R}}(\boldsymbol{k}_s, \boldsymbol{k}_s') \cdot \boldsymbol{h}_s(\boldsymbol{k}_s') = \overline{\boldsymbol{F}} : e^{i\mathscr{K}_z z} : \overline{\mathscr{G}} : \overline{\mathscr{R}} : \boldsymbol{h}_s \tag{9d}$$

where $e^{i\mathscr{K}_z z}$ is a diagonal operator, $\overline{\mathscr{G}}$ is a block diagonal operator, and $\overline{\mathscr{R}}$ is a nondiagonal operator.

Furthermore, the inverse of a diagonal operator defined in the above sense is easily found. For example, if $\mathscr{B}$ is a diagonal operator whose diagonal element is $B(\boldsymbol{k}_s)$, then the inverse of $\mathscr{B}$, i.e., $\mathscr{B}^{-1}$, is just an operator with diagonal element $1/B(\boldsymbol{k}_s)$. We can prove this quite easily because

$$\mathscr{B} : \mathscr{B}^{-1} = \int d\boldsymbol{k}_s' \delta(\boldsymbol{k}_s - \boldsymbol{k}_s') B(\boldsymbol{k}_s') \delta(\boldsymbol{k}_s' - \boldsymbol{k}_s'') \frac{1}{B(\boldsymbol{k}_s'')} = \delta(\boldsymbol{k}_s - \boldsymbol{k}_s'') = \mathscr{I}. \tag{9e}$$

B. Derivation of Field Expressions in a Homogeneous Medium

We would like to find the field expressions due to an arbitrary horizontal current sheet source located at $z = z'$ in a homogeneous medium. Starting with the scalar wave equation for the longitudinal component of the electric field,

$$(\nabla^2 + k^2) E_z(x, y, z) = 0 \tag{10}$$

the solution for $E_z(x, y, z)$ can be written as

$$E_z(x, y, z) = \frac{1}{4\pi^2} \int d\boldsymbol{k}_s e^{i\boldsymbol{k}_s \cdot \boldsymbol{r}_s} \tilde{E}_z(\boldsymbol{k}_s, z). \tag{11}$$

By requiring $E_z(x, y, z)$ to be a solution of the wave equation with a source at $z = z'$, we can deduce that

$$\tilde{E}_z(\boldsymbol{k}_s, z) = \pm \tilde{e}(\boldsymbol{k}_s) e^{ik_z|z-z'|} \qquad \text{for } z \gtrless z'. \tag{12}$$

The $\pm$ sign comes about because a horizontal current sheet gives rise to E_z that is antisymmetric about $z = z'$. Thus, $E_z(x, y, z)$ can be written as

$$E_z(x, y, z) = \pm \frac{1}{4\pi^2} \int d\boldsymbol{k}_s e^{i\boldsymbol{k}_s \cdot \boldsymbol{r}_s} \tilde{e}(\boldsymbol{k}_s) e^{ik_z|z-z'|} \qquad \text{for } z \gtrless z'. \tag{13}$$

Similarly, we can express $H_z(x, y, z)$ as

$$H_z(x, y, z) = \frac{1}{4\pi^2} \int d\boldsymbol{k}_s e^{i\boldsymbol{k}_s \cdot \boldsymbol{r}_s} \tilde{h}(\boldsymbol{k}_s) e^{ik_z|z-z'|}. \tag{14}$$

The transverse-field components are derived from the longitudinal components as

$$\boldsymbol{E}_s(\boldsymbol{r}) = \frac{1}{4\pi^2} \int d\boldsymbol{k}_s e^{i\boldsymbol{k}_s \cdot \boldsymbol{r}_s} \frac{1}{k_s^2} \left(-k_z \boldsymbol{k}_s \tilde{e}(\boldsymbol{k}_s) - \omega\mu \boldsymbol{k}_s \times \hat{z} \tilde{h}(\boldsymbol{k}_s) \right) e^{ik_z|z-z'|} \qquad \text{for } z \gtrless z' \tag{15a}$$

and

$$\boldsymbol{H}_s(\boldsymbol{r}) = \frac{1}{4\pi^2} \int d\boldsymbol{k}_s e^{i\boldsymbol{k}_s \cdot \boldsymbol{r}_s} \frac{1}{k_s^2} \left(\mp k_z \boldsymbol{k}_s \tilde{h}(\boldsymbol{k}_s) \pm \omega\epsilon \boldsymbol{k}_s \times \hat{z} \tilde{e}(\boldsymbol{k}_s) \right) e^{ik_z|z-z'|} \qquad \text{for } z \gtrless z' \tag{15b}$$

by applying the relations given in [24, sec. 3.6]. Equations (15a) and (15b) can be written more compactly as

$$\boldsymbol{E}_s(\boldsymbol{r}) = \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \frac{1}{4\pi^2} \int d\boldsymbol{k}_s \overline{\boldsymbol{F}}(\boldsymbol{k}_s, \boldsymbol{r}_s) \cdot e^{ik_z|z-z'|} \boldsymbol{e}_s(\boldsymbol{k}_s) \tag{16a}$$

and

$$\boldsymbol{H}_s(\boldsymbol{r}) = \begin{bmatrix} H_y \\ -H_x \end{bmatrix} = \frac{\operatorname{sgn}(z-z')}{4\pi^2} \int d\boldsymbol{k}_s \overline{\boldsymbol{F}}(\boldsymbol{k}_s, \boldsymbol{r}_s) \cdot e^{ik_z|z-z'|} \boldsymbol{h}_s(\boldsymbol{k}_s) \tag{16b}$$

where $\operatorname{sgn}(z)$ is the sign of z, and

$$\boldsymbol{e}_s(\boldsymbol{k}_s) = \begin{bmatrix} -\dfrac{k_z \tilde{e}(\boldsymbol{k}_s)}{k_s} \\ -\dfrac{\omega\mu \tilde{h}(\boldsymbol{k}_s)}{k_s} \end{bmatrix} \qquad \boldsymbol{h}_s(\boldsymbol{k}_s) = \begin{bmatrix} -\dfrac{\omega\epsilon \tilde{e}(\boldsymbol{k}_s)}{k_s} \\ -\dfrac{k_z \tilde{h}(\boldsymbol{k}_s)}{k_s} \end{bmatrix} \tag{17}$$

and $\overline{\boldsymbol{F}}(\boldsymbol{k}_s, \boldsymbol{r}_s)$ is given by

$$\overline{\boldsymbol{F}}(\boldsymbol{k}_s, \boldsymbol{r}_s) = \frac{e^{i\boldsymbol{k}_s \cdot \boldsymbol{r}_s}}{k_s} \begin{bmatrix} k_x & k_y \\ k_y & -k_x \end{bmatrix}. \tag{18}$$

equations (16a) and (16b) are vector Fourier transform integrals [1]. A vector Fourier transform (VFT) pair exists through the relationships

$$J_s(r_s) = \frac{1}{4\pi^2}\int dk_s \bar{F}(k_s, r_s)\cdot \tilde{J}_s(k_s) \quad (19a)$$

$$\tilde{J}_s(k_s) = \int dr_s \bar{F}(k_s, -r_s)\cdot J_s(r_s). \quad (19b)$$

The discontinuity of H_s at $z = z'$ in (16b) is due to the presence of a current sheet at $z = z'$. Hence, we can write

$$J_s(r_s) = \begin{bmatrix} J_x \\ J_y \end{bmatrix} = -\frac{1}{4\pi^2}\int dk_s \bar{F}(k_s, r_s)\cdot 2h_s(k_s). \quad (20)$$

Using VFT, we deduce that

$$\tilde{J}_s(k_s) = -2h_s(k_s) \quad (21)$$

where $\tilde{J}_s(k_s)$ is the VFT of $J_s(r_s)$. Defining the dyadic Green's function $\bar{G}(k_s)$ as

$$e_s(k_s) = -\bar{G}(k_s)\cdot 2h_s(k_s) = \bar{G}(k_s)\cdot\tilde{J}_s(k_s) \quad (22)$$

(16a) and (16b) can be rewritten as

$$E_s(r) = \frac{1}{4\pi^2}\int dk_s \bar{F}(k_s, r_s)\cdot e^{ik_z|z-z'|}\bar{G}(k_s)\cdot\tilde{J}_s(k_s) \quad (23a)$$

$$H_s(r) = -\frac{\operatorname{sgn}(z-z')}{8\pi^2}\int dk_s \bar{F}(k_s, r_s)\cdot e^{ik_z|z-z'|}\tilde{J}_s(k_s) \quad (23b)$$

where

$$\bar{G}(k_s) = -\frac{1}{2}\begin{bmatrix} \frac{k_z}{\omega\epsilon} & 0 \\ 0 & \frac{\omega\mu}{k_z} \end{bmatrix}. \quad (24)$$

The field expressions of (23) form the basic convention that we will use in the following sections to derive some reflection operators and guidance conditions. However, before using these expressions in other sections, we will take a final step to cast them in a more compact form, namely, in double dot product notation:

$$E_s(r) = \bar{F}(r_s) : e^{i\mathscr{K}_z|z-z'|} : \mathscr{G} : \tilde{J}_s \quad (25a)$$

$$H_s(r) = -\frac{\operatorname{sgn}(z-z')}{2}\tilde{F}(r_s) : e^{i\mathscr{K}_z|z-z'|} : \tilde{J}_s. \quad (25b)$$

In the above, we have absorbed $1/4\pi^2$ in $\tilde{J}_s$. We shall do the same for the rest of the paper. Note how compact the expressions are after using the new notations. In the expressions, we can also see waves propagating away from $z = z'$ via the propagator $e^{i\mathscr{K}_z|z-z'|}$. The above looks like a separation of variables expression, but in higher dimensions.

III. Reflection and Transmission Operators

A. A Strip or a Disk in a Homogeneous Medium

In this section, we will derive the reflection and transmission operators characterizing the scattering by a thin, infinitely conducting strip or disk of finite size. The derivation for the disk case is very similar to that for the strip case. In the case of a strip, we may reduce our double integrals to single integrals, since we only need a one-dimensional Fourier transform. In the rest of the paper, we may neglect to mention specifically the disk case, but it is understood from the context that the derivation for the disk case is very similar.

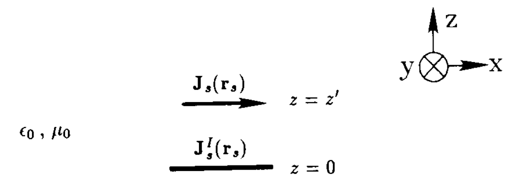

Fig. 1. Strip or disk in a homogeneous medium.

In deriving the reflection operator for the strip, we illuminate the strip by a source as shown in Fig. 1. We may assume that the illuminating source is from a horizontal current sheet as described in Section II. Except for the strip and the source, the medium is homogeneous. The strip is located at $z = 0$. When the strip is illuminated by a source, induced currents on the strip cause reradiation or scattering. In the manner of (25), with sources at $z = z'$ and $z = 0$, we can write the total field for all z as

$$E_s(r) = \bar{F}(r_s) : \left(e^{i\mathscr{K}_z|z-z'|} : \mathscr{G} : \tilde{J}_s + e^{i\mathscr{K}_z|z|} : \mathscr{G} : \tilde{J}_s^I\right) \quad (26a)$$

$$H_s(r) = -\tfrac{1}{2}\tilde{F}(r_s) : \left(\operatorname{sgn}(z-z')e^{i\mathscr{K}_z|z-z'|} : \tilde{J}_s + \operatorname{sgn}(z)e^{i\mathscr{K}_z|z|} : \tilde{J}_s^I\right). \quad (26b)$$

$\tilde{J}_s^I(k_s)$ in the above is the VFT of the induced current on the illuminated strip. We can expand the induced surface current $J_s^I(r_s)$ on the strip as

$$J_s^I(r_s) = \sum_n \bar{J}_{sn}(r_s)\cdot a_n \quad (27)$$

where $\bar{J}_{sn}(r_s)$ may be of the form

$$\bar{J}_{sn}(r_s) = \begin{bmatrix} J_{xn}(r_s) & 0 \\ 0 & J_{yn}(r_s) \end{bmatrix} \quad \text{and} \quad a_n = \begin{bmatrix} a_n \\ b_n \end{bmatrix}. \quad (28)$$

Alternatively, we can write (27) more compactly as

$$J_s^I(r_s) = f^t(r_s)\cdot A \quad (29)$$

where

$$f^t(r_s) = [\cdots \bar{J}_{sn}(r_s) \cdots] \quad \text{and} \quad A = \begin{bmatrix} \vdots \\ a_n \\ \vdots \end{bmatrix}. \quad (30)$$

Taking the VFT of (29), we have

$$\tilde{J}_s^I(k_s) = \tilde{f}^t(k_s)\cdot A. \quad (31)$$

Substituting (31) into (26a), we have

$$E_s(r) = \bar{F}(r_s) : \left(e^{i\mathscr{K}_z|z|} : \mathscr{G} : \tilde{f}^t\cdot A + e^{i\mathscr{K}_z|z-z'|} : \mathscr{G} : \tilde{J}_s\right). \quad (32)$$

We require that $E_s(r_s, z = 0) = 0$ on the strip. Setting $z = 0$ in (32), multiplying by a vector of testing functions, $f_t(r_s)$, and integrating over the strip, we have

$$\int d\boldsymbol{r}_s\, \boldsymbol{f}_t(\boldsymbol{r}_s)\cdot \bar{\boldsymbol{F}}(\boldsymbol{r}_s):\left(e^{i\mathcal{K}_z z'}:\bar{\mathcal{G}}:\tilde{\boldsymbol{J}}_s+\bar{\mathcal{G}}:\tilde{\boldsymbol{f}}^t\cdot \boldsymbol{A}\right)=0 \tag{33}$$

where

$$\boldsymbol{f}_t(\boldsymbol{r}_s)=\begin{bmatrix}\vdots\\ J^t_{tm}(\boldsymbol{r}_s)\\ \vdots\end{bmatrix}. \tag{34}$$

Defining

$$\tilde{\boldsymbol{f}}_t(\boldsymbol{k}_s)=\int d\boldsymbol{r}_s\, \boldsymbol{f}_t(\boldsymbol{r}_s)\cdot \bar{\boldsymbol{F}}(\boldsymbol{k}_s,\boldsymbol{r}_s) \tag{35}$$

(33) becomes

$$\tilde{\boldsymbol{f}}_t(\boldsymbol{k}_s):\left(e^{i\mathcal{K}_z z'}:\bar{\mathcal{G}}:\tilde{\boldsymbol{J}}_s+\bar{\mathcal{G}}:\tilde{\boldsymbol{f}}^t\cdot \boldsymbol{A}\right)=0. \tag{36}$$

Solving for $\boldsymbol{A}$, we have

$$\boldsymbol{A}=-\bar{\Gamma}^{-1}\cdot \tilde{\boldsymbol{f}}_t:\bar{\mathcal{G}}:e^{i\mathcal{K}_z z'}:\tilde{\boldsymbol{J}}_s \tag{37}$$

where

$$\bar{\Gamma}=\tilde{\boldsymbol{f}}_t:\bar{\mathcal{G}}:\tilde{\boldsymbol{f}}^t. \tag{38}$$

In the above, $\bar{\Gamma}$ is a matrix with discrete indexing. Its size depends on the length of the basis function and testing function vectors we have in (30) and (34). In theory, it should be an infinite size matrix. However, from a practical viewpoint, we need to truncate this matrix. This is equivalent to choosing vectors of length N in both (30) and (34). In this case, $\bar{\Gamma}$ will be a $2N\times 2N$ square matrix which is easily invertible on a computer. Note that the same result would have been arrived at if we applied Galerkin's method [25] to the integral equation obtained by requiring that $\boldsymbol{E}_s(\boldsymbol{r}_s, z=0)=0$ on the strip, except that our notation is more symbolic and compact.

Since $\tilde{\boldsymbol{J}}^I_s(\boldsymbol{k}_s)$ or $\boldsymbol{A}$ is linearly proportional to $\tilde{\boldsymbol{J}}_s(\boldsymbol{k}_s)$, we can rewrite (26) as

$$\boldsymbol{E}_s(\boldsymbol{r})=\bar{\boldsymbol{F}}(\boldsymbol{r}_s):\left(e^{i\mathcal{K}_z|z-z'|}+e^{i\mathcal{K}_z|z|}:\bar{\mathcal{R}}:e^{i\mathcal{K}_z z'}\right):\bar{\mathcal{G}}:\tilde{\boldsymbol{J}}_s \tag{39a}$$

$$\boldsymbol{H}_s(\boldsymbol{r})=-\tfrac{1}{2}\bar{\boldsymbol{F}}(\boldsymbol{r}_s):\bar{\mathcal{G}}^{-1}:\left(\operatorname{sgn}(z-z')e^{i\mathcal{K}_z|z-z'|}+\operatorname{sgn}(z)e^{i\mathcal{K}_z|z|}:\bar{\mathcal{R}}:e^{i\mathcal{K}_z z'}\right):\bar{\mathcal{G}}:\tilde{\boldsymbol{J}}_s \tag{39b}$$

where

$$\bar{\mathcal{R}}=-\bar{\mathcal{G}}:\tilde{\boldsymbol{f}}^t\cdot\bar{\Gamma}^{-1}\cdot\tilde{\boldsymbol{f}}_t \tag{40}$$

is the reflection operator for the electric field for the finite-width strip. It is defined as the "ratio" of the reflected electric field to the incident electric field $z=0$.

When $z<0$, we can rewrite (39) as

$$\boldsymbol{E}_s(\boldsymbol{r})=\bar{\boldsymbol{F}}(\boldsymbol{r}_s):e^{-i\mathcal{K}_z z}:\bar{\mathcal{T}}:\bar{\mathcal{G}}:e^{i\mathcal{K}_z z'}:\tilde{\boldsymbol{J}}_s \tag{41a}$$

$$\boldsymbol{H}_s(\boldsymbol{r})=\tfrac{1}{2}\bar{\boldsymbol{F}}(\boldsymbol{r}_s):\bar{\mathcal{G}}^{-1}:e^{-i\mathcal{K}_z z}:\bar{\mathcal{T}}:e^{i\mathcal{K}_z z'}:\bar{\mathcal{G}}:\tilde{\boldsymbol{J}}_s \tag{41b}$$

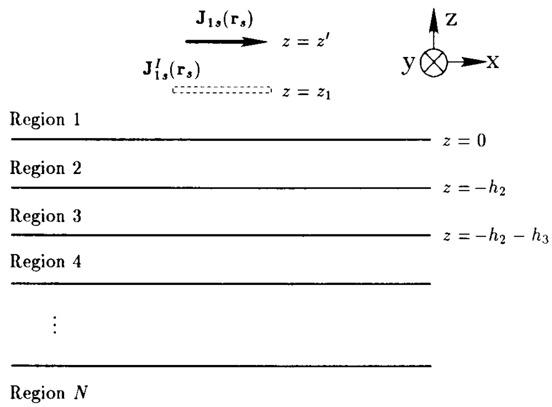

Fig. 2. Strip or disk over a layered medium.

where

$$\bar{\mathcal{T}}=\bar{\mathcal{I}}+\bar{\mathcal{R}} \tag{42}$$

is the transmission operator of the strip. $\bar{\mathcal{R}}$ and $\bar{\mathcal{T}}$ are, in general, nondiagonal operators.

B. A Strip or a Disk over a Layered Medium

If a source is over a layered medium as shown in Fig. 2, the source generates a spectrum of plane waves that will be reflected by the layered medium. The TM waves will be reflected according to the TM reflection coefficients while the TE waves will be reflected according to the TE reflection coefficients. We can easily show that the fields above the layered medium can be written as

$$\boldsymbol{E}_{1s}(\boldsymbol{r})=\bar{\boldsymbol{F}}(\boldsymbol{r}_s):\left(e^{i\mathcal{K}_{1z}|z-z'|}+e^{i\mathcal{K}_{1z}z}:\tilde{\bar{\mathcal{R}}}_{12}:e^{i\mathcal{K}_{1z}z'}\right):\bar{\mathcal{G}}_1:\tilde{\boldsymbol{J}}_{1s},\qquad z>0 \tag{43a}$$

$$\boldsymbol{H}_{1s}(\boldsymbol{r})=-\tfrac{1}{2}\bar{\boldsymbol{F}}(\boldsymbol{r}_s):\left(\operatorname{sgn}(z-z')e^{i\mathcal{K}_{1z}|z-z'|}+e^{i\mathcal{K}_{1z}z}:\tilde{\bar{\mathcal{R}}}_{12}:e^{i\mathcal{K}_{1z}z'}\right):\tilde{\boldsymbol{J}}_{1s},\qquad z>0. \tag{43b}$$

The subscript 1 denotes a quantity associated with region 1. The above is of the form of (39), but the reflection operators are diagonal operators expressible in terms of generalized reflection coefficients. In particular, $\tilde{\bar{\mathcal{R}}}_{12}$ is the operator representation of the matrix reflection coefficient

$$\bar{\boldsymbol{R}}_{12}=\begin{bmatrix}-\tilde{R}^{\mathrm{TM}}_{12} & 0\\ 0 & \tilde{R}^{\mathrm{TE}}_{12}\end{bmatrix}. \tag{44}$$

We shall present the derivation of $\bar{\boldsymbol{R}}_{12}$ in the Appendix. When the layered medium reduces to a half-space, $\tilde{R}^{\mathrm{TM}}_{12}$ and $\tilde{R}^{\mathrm{TE}}_{12}$ are just the Fresnel reflection coefficients.

If we have a strip on top of the layered medium located at $z=0$ as shown with the dotted lines in Fig. 2, the induced currents on the strip will generate a field in addition to that of (43). Consequently, for a strip on top of the layered medium illuminated by the source field given by (43), we have

$$\boldsymbol{E}_{1s}(\boldsymbol{r}) = \overline{\boldsymbol{F}}(\boldsymbol{r}_s):\left\{\left(e^{i\mathscr{K}_{1z}|z-z'|} + e^{i\mathscr{K}_{1z}z}:\tilde{\overline{\overline{\mathscr{R}}}}_{12}:e^{i\mathscr{K}_{1z}z'}\right):\overline{\mathscr{G}}_1:\tilde{\boldsymbol{J}}_{1s} + \left(e^{i\mathscr{K}_{1z}|z-z_1|} + e^{i\mathscr{K}_{1z}z}:\tilde{\overline{\overline{\mathscr{R}}}}_{12}:e^{i\mathscr{K}_{1z}z_1}\right):\overline{\mathscr{G}}_1:\tilde{\boldsymbol{J}}^I_{1s}\right\}, \quad z>0 \quad (45a)$$

$$\boldsymbol{H}_{1s}(\boldsymbol{r}) = -\tfrac{1}{2}\overline{\boldsymbol{F}}(\boldsymbol{r}_s):\left\{\left(\operatorname{sgn}(z-z')e^{i\mathscr{K}_{1z}|z-z'|} + e^{i\mathscr{K}_{1z}z}:\tilde{\overline{\overline{\mathscr{R}}}}_{12}:e^{i\mathscr{K}_{1z}z'}\right):\tilde{\boldsymbol{J}}_{1s} + \left(\operatorname{sgn}(z)e^{i\mathscr{K}_{1z}|z-z_1|} + e^{i\mathscr{K}_{1z}z}:\tilde{\overline{\overline{\mathscr{R}}}}_{12}:e^{i\mathscr{K}_{1z}z_1}\right):\tilde{\boldsymbol{J}}^I_{1s}\right\}, \quad z>0. \quad (45b)$$

In the above, $\tilde{\boldsymbol{J}}^I_{1s}(\boldsymbol{k}_s)$ is the unknown to be sought. The procedure of finding $\tilde{\boldsymbol{J}}^I_{1s}(\boldsymbol{k}_s)$ is similar to that of the previous subsection. Similar to (31), we let

$$\tilde{\boldsymbol{J}}^I_{1s}(\boldsymbol{k}_s) = \boldsymbol{f}^t(\boldsymbol{k}_s)\cdot\boldsymbol{A}. \quad (46)$$

Imposing the requirement that $\boldsymbol{E}_{1s}(\boldsymbol{r}_s, z=z_1) = 0$ on the strip, we have

$$\overline{\boldsymbol{F}}(\boldsymbol{r}_s):\left\{\left(e^{i\mathscr{K}_{1z}(z'-z_1)} + e^{i\mathscr{K}_{1z}z_1}:\tilde{\overline{\overline{\mathscr{R}}}}_{12}:e^{i\mathscr{K}_{1z}z'}\right):\overline{\mathscr{G}}_1:\tilde{\boldsymbol{J}}_{1s} + \left(\overline{\mathscr{I}} + e^{i\mathscr{K}_{1z}z_1}:\tilde{\overline{\overline{\mathscr{R}}}}_{12}:e^{i\mathscr{K}_{1z}z_1}\right):\overline{\mathscr{G}}_1:\boldsymbol{f}^t\cdot\boldsymbol{A}\right\} = 0. \quad (47)$$

Weighting the above by $\boldsymbol{f}_t(\boldsymbol{r}_s)$ as in (33), we arrive at

$$\boldsymbol{A} = -\overline{\Gamma}^{-1}\cdot\boldsymbol{f}_t:\left(\overline{\mathscr{I}} + e^{i\mathscr{K}_{1z}z_1}:\tilde{\overline{\overline{\mathscr{R}}}}_{12}:e^{i\mathscr{K}_{1z}z_1}\right):e^{i\mathscr{K}_{1z}(z'-z_1)}:\overline{\mathscr{G}}_1:\tilde{\boldsymbol{J}}_{1s} \quad (48)$$

where

$$\overline{\Gamma} = \boldsymbol{f}_t:\left(\overline{\mathscr{I}} + e^{i\mathscr{K}_{1z}z_1}:\tilde{\overline{\overline{\mathscr{R}}}}_{12}:e^{i\mathscr{K}_{1z}z_1}\right):\overline{\mathscr{G}}_1:\boldsymbol{f}^t. \quad (49)$$

Note the form of $\overline{\Gamma}$ is very similar to that of (38) except for the modification in the $\overline{\mathscr{G}}$. This new form of $\overline{\mathscr{G}}$ can be thought of as the new Green's function that the induced current on the strip is now radiating with. Making use of (46) and (48) in (45), we have

$$\boldsymbol{E}_{1s}(\boldsymbol{r}) = \overline{\boldsymbol{F}}(\boldsymbol{r}_s):\left\{\left(e^{i\mathscr{K}_{1z}|z-z'|} + e^{i\mathscr{K}_{1z}z}:\tilde{\overline{\overline{\mathscr{R}}}}_{12}:e^{i\mathscr{K}_{1z}z'}\right) - e^{i\mathscr{K}_{1z}(z-z_1)}:\left(\overline{\mathscr{I}} + e^{i\mathscr{K}_{1z}z_1}:\tilde{\overline{\overline{\mathscr{R}}}}_{12}:e^{i\mathscr{K}_{1z}z_1}\right):\overline{\mathscr{G}}_1:\boldsymbol{f}^t\cdot\overline{\Gamma}^{-1}\cdot\boldsymbol{f}_t:\left(\overline{\mathscr{I}} + e^{i\mathscr{K}_{1z}z_1}:\tilde{\overline{\overline{\mathscr{R}}}}_{12}:e^{i\mathscr{K}_{1z}z_1}\right):e^{i\mathscr{K}_{1z}(z'-z_1)}\right\}:\overline{\mathscr{G}}_1:\tilde{\boldsymbol{J}}_{1s}; \quad z>z_1. \quad (50)$$

We can rewrite the above as

$$\boldsymbol{E}_{1s}(\boldsymbol{r}) = \overline{\boldsymbol{F}}(\boldsymbol{r}_s):\left(e^{i\mathscr{K}_{1z}|z-z'|} + e^{i\mathscr{K}_{1z}(z-z_1)}:\tilde{\overline{\mathscr{R}}}:e^{i\mathscr{K}_{1z}(z'-z_1)}\right):\overline{\mathscr{G}}_1:\tilde{\boldsymbol{J}}_{1s}, \quad z>z_1 \quad (51)$$

where

$$\tilde{\overline{\mathscr{R}}} = e^{i\mathscr{K}_{1z}z_1}:\tilde{\overline{\overline{\mathscr{R}}}}_{12}:e^{i\mathscr{K}_{1z}z_1} - \left(\overline{\mathscr{I}} + e^{i\mathscr{K}_{1z}z_1}:\tilde{\overline{\overline{\mathscr{R}}}}_{12}:e^{i\mathscr{K}_{1z}z_1}\right):\overline{\mathscr{G}}_1:\boldsymbol{f}^t\cdot\overline{\Gamma}^{-1}\cdot\boldsymbol{f}_t:\left(\overline{\mathscr{I}} + e^{i\mathscr{K}_{1z}z_1}:\tilde{\overline{\overline{\mathscr{R}}}}_{12}:e^{i\mathscr{K}_{1z}z_1}\right) \quad (52)$$

is the reflection operator relating the upgoing wave to the downgoing wave in region 1 defined at $z=0$. For the layered medium case, $\tilde{\overline{\overline{\mathscr{R}}}}_{12}$ is diagonal. However, the above derivation is also valid when $\tilde{\overline{\overline{\mathscr{R}}}}_{12}$ is nondiagonal. This is the case when the strip is on top of another strip, whose reflection operator is derived in subsection A. Then (52) can be used to find the reflection operator for two strips which are on top of each other. In fact, given the reflection operator for the two-strip case, we can use (52) again to find the reflection operator of the three-strip case. Hence, (52) forms a recursive relation from which we can find the reflection operator of, and hence the scattering from, N strips in a homogeneous medium.

C. A Strip or a Disk Embedded in a Layered Medium

Next, we would like to find the reflection operator as defined at $z=d_1$ due to a strip or a disk embedded in a layered medium as shown in Fig. 3. Just as in subsection B, we can consider first the scattering of the field by the layered medium in the absence of the strip or disk. In this case, the field in region 1 is

$$\boldsymbol{E}^O_{1s}(\boldsymbol{r}) = \overline{\boldsymbol{F}}(\boldsymbol{r}_s):\left(e^{i\mathscr{K}_{1z}|z-z'|} + e^{i\mathscr{K}_{1z}(z-d_1)}:\tilde{\overline{\overline{\mathscr{R}}}}_{12}:e^{i\mathscr{K}_{1z}(z'-d_1)}\right):\overline{\mathscr{G}}_1:\tilde{\boldsymbol{J}}_{1s} \quad (53a)$$

$$\boldsymbol{H}^O_{1s}(\boldsymbol{r}) = -\tfrac{1}{2}\overline{\boldsymbol{F}}(\boldsymbol{r}_s):\left(\operatorname{sgn}(z-z')e^{i\mathscr{K}_{1z}|z-z'|} + e^{i\mathscr{K}_{1z}(z-d_1)}:\tilde{\overline{\overline{\mathscr{R}}}}_{12}:e^{i\mathscr{K}_{1z}(z'-d_1)}\right):\tilde{\boldsymbol{J}}_{1s} \quad (53b)$$

where $\tilde{\overline{\overline{\mathscr{R}}}}_{12}$ is the generalized reflection operator for the layered medium. In this case, it is diagonal. The field in region 2 in the absence of the strip or disk can be written as

$$\boldsymbol{E}^O_{2s}(\boldsymbol{r}) = \overline{\boldsymbol{F}}(\boldsymbol{r}_s):\left(e^{-i\mathscr{K}_{2z}(z-d_1')} + e^{i\mathscr{K}_{2z}(z-d_2)}:\tilde{\overline{\overline{\mathscr{R}}}}_{23}:e^{i\mathscr{K}_{2z}(d_1'-d_2)}\right):\tilde{\overline{\overline{\mathscr{T}}}}_{12}:e^{i\mathscr{K}_{1z}(z'-d_1)}:\overline{\mathscr{G}}_1:\tilde{\boldsymbol{J}}_{1s} \quad (54a)$$

$$\boldsymbol{H}^O_{2s}(\boldsymbol{r}) = -\tfrac{1}{2}\overline{\boldsymbol{F}}(\boldsymbol{r}_s):\overline{\mathscr{G}}_2^{-1}:\left(-e^{-i\mathscr{K}_{2z}(z-d_1')} + e^{i\mathscr{K}_{2z}(z-d_2)}:\tilde{\overline{\overline{\mathscr{R}}}}_{23}:e^{i\mathscr{K}_{2z}(d_1'-d_2)}\right):\tilde{\overline{\overline{\mathscr{T}}}}_{12}:e^{i\mathscr{K}_{1z}(z'-d_1)}:\overline{\mathscr{G}}_1:\tilde{\boldsymbol{J}}_{1s}. \quad (54b)$$

In the above, $\tilde{\overline{\overline{\mathscr{T}}}}_{12}$ is a generalized transmission operator that transmits a downgoing wave from region 1 to region 2. We will discuss the derivation of $\tilde{\overline{\overline{\mathscr{T}}}}_{12}$ in the Appendix.

In the presence of the strip, induced currents on the strip will reradiate. The field in region 2 due to the strip is

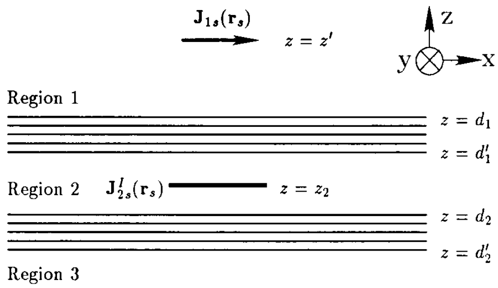

Fig. 3. Strip or disk in a layered medium.

then

$$\boldsymbol{E}^S_{2s}(\boldsymbol{r}) = \overline{\boldsymbol{F}}(\boldsymbol{r}_s) : \Big(e^{i\mathscr{K}_{2z}|z-z_2|} + e^{i\mathscr{K}_{2z}(z-d_2)} : \overline{\mathscr{U}}_2 : e^{i\mathscr{K}_{2z}(z_2-d_2)} + e^{-i\mathscr{K}_{2z}(z-d_1')} : \overline{\mathscr{D}}_2 : e^{i\mathscr{K}_{2z}(d_1'-z_2)} \Big) : \overline{\mathscr{G}}_2 : \tilde{\boldsymbol{J}}^I_{2s} \quad (55a)$$

$$\boldsymbol{H}^S_{2s}(\boldsymbol{r}) = -\tfrac{1}{2}\overline{\boldsymbol{F}}(\boldsymbol{r}_s) : \Big(\operatorname{sgn}(z - z_2) e^{i\mathscr{K}_{2z}|z-z_2|} + e^{i\mathscr{K}_{2z}(z-d_2)} : \overline{\mathscr{U}}_2 : e^{i\mathscr{K}_{2z}(z_2-d_2)} - e^{i\mathscr{K}_{2z}(z-d_1')} : \overline{\mathscr{D}}_2 : e^{i\mathscr{K}_{2z}(d_1'-z_2)} \Big) : \tilde{\boldsymbol{J}}^I_{2s}. \quad (55b)$$

The derivations of $\overline{\mathscr{U}}_2$ and $\overline{\mathscr{D}}_2$ will be discussed in the Appendix. If the medium is homogeneous below the strip, then $\overline{\mathscr{U}}_2 = 0$ and $\overline{\mathscr{D}}_2 = \overline{\overline{\mathscr{R}}}_{21}$ which is the generalized reflection operator for the layered medium above the strip.

The total field in region 2 is given by the sum of (54) and (55). Again, we require that the total $\boldsymbol{E}_{2s}(\boldsymbol{r}) = 0$ on the strip in region 2. Going through the derivation as before, we expand the surface current density $\boldsymbol{J}^I_{2s}(\boldsymbol{r}_s)$ at $z = z_2$ in terms of $\boldsymbol{f}^t_2(\boldsymbol{r}_s)\cdot\boldsymbol{A}$. Then, after taking the VFT, we find that

$$\tilde{\boldsymbol{J}}^I_{2s}(\boldsymbol{k}_s) = \tilde{\boldsymbol{f}}^t_2(\boldsymbol{k}_s)\cdot\boldsymbol{A}. \quad (56)$$

Requiring that $\boldsymbol{E}_{2s}(\boldsymbol{r}) = 0$ on the strip, we deduce that

$$\tilde{\boldsymbol{J}}^I_{2s} = -\tilde{\boldsymbol{f}}^t_2 \cdot \overline{\Gamma}_2^{-1} \cdot \tilde{\boldsymbol{f}}_{2t} : \Big(e^{-i\mathscr{K}_{2z}(z_2-d_1')} + e^{i\mathscr{K}_{2z}(z_2-d_2)} : \overline{\overline{\mathscr{R}}}_{23} : e^{i\mathscr{K}_{2z}(d_1'-d_2)} \Big) : \overline{\overline{\mathscr{T}}}_{12} : e^{i\mathscr{K}_{1z}(z'-d_1)} : \overline{\mathscr{G}}_1 : \tilde{\boldsymbol{J}}_{1s} \quad (57)$$

where

$$\overline{\Gamma}_2 = \tilde{\boldsymbol{f}}_{2t} : \Big(\mathscr{I} + e^{i\mathscr{K}_{2z}(z_2-d_2)} : \overline{\mathscr{U}}_2 : e^{i\mathscr{K}_{2z}(z_2-d_2)} + e^{-i\mathscr{K}_{2z}(z_2-d_1')} : \overline{\mathscr{D}}_2 : e^{i\mathscr{K}_{2z}(d_1'-z_2)} \Big) : \overline{\mathscr{G}}_2 : \tilde{\boldsymbol{f}}^t_2. \quad (57a)$$

The form of $\overline{\Gamma}_2$ is again very similar to that of (38) except for the modification of $\overline{\mathscr{G}}$. This new $\overline{\mathscr{G}}$ is now the new Green's function that the current on the strip is radiating with.

The field in region 1 due to the induced current on the embedded strip is a consequence of the transmission of the upgoing wave in region 2. Hence, we can identify the upgoing wave in region 2 and multiply it by a transmission operator $\overline{\overline{\mathscr{T}}}_{21}$ that transmits an upgoing wave from region 2 to region 1. Consequently, the field in region 1 due to the strip in region 2 is

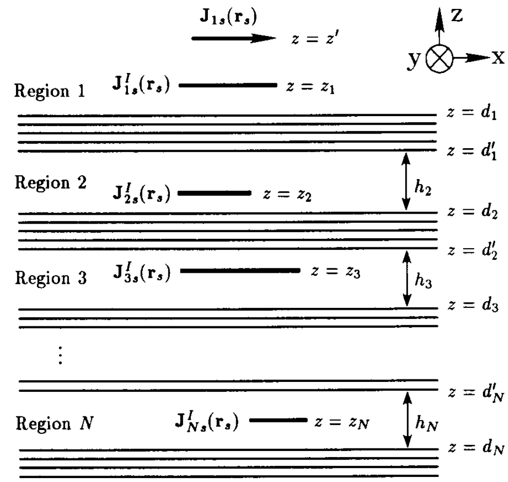

Fig. 4. N strips or disks in a layered medium.

$$\boldsymbol{E}^S_{1s}(\boldsymbol{r}) = \overline{\boldsymbol{F}}(\boldsymbol{r}_s) : e^{i\mathscr{K}_{1z}(z-d_1)} : \overline{\overline{\mathscr{T}}}_{21} : \Big(e^{i\mathscr{K}_{2z}(d_1'-z_2)} + e^{i\mathscr{K}_{2z}(d_1'-d_2)} : \overline{\mathscr{U}}_2 : e^{i\mathscr{K}_{2z}(z_2-d_2)} \Big) : \overline{\mathscr{G}}_2 : \tilde{\boldsymbol{J}}^I_{2s} \quad (58a)$$

$$\boldsymbol{H}^S_{1s}(\boldsymbol{r}) = -\tfrac{1}{2}\overline{\boldsymbol{F}}(\boldsymbol{r}_s) : \overline{\mathscr{G}}_1^{-1} : e^{i\mathscr{K}_{1z}(z-d_1)} : \overline{\overline{\mathscr{T}}}_{21} : \Big(e^{i\mathscr{K}_{2z}(d_1'-z_2)} + e^{i\mathscr{K}_{2z}(d_1'-d_2)} : \overline{\mathscr{U}}_2 : e^{i\mathscr{K}_{2z}(z_2-d_2)} \Big) : \overline{\mathscr{G}}_2 : \tilde{\boldsymbol{J}}^I_{2s}. \quad (58b)$$

The total field in region 1 is the sum of (53) and (58). From (57), we see that $\tilde{\boldsymbol{J}}^I_{2s}$ is of the form

$$\tilde{\boldsymbol{J}}^I_{2s} = \overline{\mathscr{P}} : e^{i\mathscr{K}_{1z}(z'-d_1)} : \overline{\mathscr{G}}_1 : \tilde{\boldsymbol{J}}_{1s}. \quad (59)$$

We can define a reflection operator relating the upgoing wave to the downgoing wave in region 1 when the subsurface medium contains a single embedded strip. It is

$$\overline{\mathscr{R}} = \overline{\overline{\mathscr{R}}}_{12} - \overline{\overline{\mathscr{T}}}_{21} : \Big(e^{i\mathscr{K}_{2z}(d_1'-z_2)} + e^{i\mathscr{K}_{2z}(d_1'-d_2)} : \overline{\mathscr{U}}_2 : e^{i\mathscr{K}_{2z}(z_2-d_2)} \Big) : \overline{\mathscr{G}}_2 : \tilde{\boldsymbol{f}}^t_2 \cdot \overline{\Gamma}_2^{-1} \cdot \tilde{\boldsymbol{f}}_{2t} : \Big(e^{-i\mathscr{K}_{2z}(z_2-d_1')} + e^{i\mathscr{K}_{2z}(z_2-d_2)} : \overline{\overline{\mathscr{R}}}_{23} : e^{i\mathscr{K}_{2z}(d_1'-d_2)} \Big) : \overline{\overline{\mathscr{T}}}_{12}. \quad (60)$$

The above derivation can be generalized easily to the case of N strips of disks embedded in a layered medium as shown in Fig. 4. It can also be generalized to study the scattering of a slot or an aperture. We have done these in [26].

IV. Conclusions

We have developed a new notation, together with vector Fourier transforms, which can be used for studying the scattering of waves from strips or disks. This new notation allows us to write complicated expressions compactly and yet not obscure the underlying physics of the problems.

We have used the new notation to derive and describe reflection and transmission operators for certain complex scattering problems by strips or disks. We first derived the reflection and transmission operators for a single disk or strip. Then we generalized it to the case of deriving the reflection operator of a single strip or disk on top of a reflecting medium. We discussed how the case of N strips or disks in a homogeneous medium can be computed using such a formalism. We next discussed how the reflection operator of an embedded strip or disk can be similarly derived. The result can be generalized to the case of scattering from N embedded strips or disks [26], which may find applications in high-speed circuitry in computer technology or in calculating the radar scattering cross section of frequency-selective surfaces. All our reflection and transmission operators thus derived are computable. They can be physically interpreted and hence used to elucidate the underlying scattering processes.

Appendix

A. Reflection and Transmission Operators for a Layered Medium

When we have a source in medium 1 as shown in Fig. 2, we can write down the expression for the field in region 1 as in (43a), viz.,

$$\boldsymbol{E}_{1s}(\boldsymbol{r}) = \bar{\boldsymbol{F}}(\boldsymbol{r}_s) : \left(e^{i\mathscr{K}_{1z}|z-z'|} + e^{i\mathscr{K}_{1z}z} : \tilde{\bar{\mathscr{R}}}_{12} : e^{i\mathscr{K}_{1z}z'}\right) : \bar{\mathscr{G}}_1 : \tilde{\boldsymbol{J}}_{1s}, \quad z > 0 \quad \text{(A1)}$$

where $\tilde{\bar{\mathscr{R}}}_{12}$ is the generalized reflection operator including subsurface reflections. The field in region 2 can be written as

$$\boldsymbol{E}_{2s}(\boldsymbol{r}) = \bar{\boldsymbol{F}}(\boldsymbol{r}_s) : \left(e^{-i\mathscr{K}_{2z}z} + e^{i\mathscr{K}_{2z}(z+h_2)} : \tilde{\bar{\mathscr{R}}}_{23} : e^{i\mathscr{K}_{2z}h_2}\right) : \boldsymbol{A}_2 \quad \text{(A2)}$$

where $\tilde{\bar{\mathscr{R}}}_{23}$ is a generalized reflection operator including subsurface reflections. We have required that the upgoing wave be related to the downgoing wave in region 2 by the reflection operator $\tilde{\bar{\mathscr{R}}}_{23}$. $\boldsymbol{A}_2$ is the amplitude of the downgoing wave in region 2. However, we know that the downgoing wave in region 2 is a consequence of a transmission of the downgoing wave in region 1 plus the reflection of the upgoing wave in region 2. Therefore,

$$\boldsymbol{A}_2 = \bar{\mathscr{T}}_{12} : e^{i\mathscr{K}_{1z}z'} : \bar{\mathscr{G}}_1 : \tilde{\boldsymbol{J}}_{1s} + \bar{\mathscr{R}}_{21} : \tilde{\bar{\mathscr{R}}}_{23} \cdot e^{i2\mathscr{K}_{2z}h_2} : \boldsymbol{A}_2 \quad \text{(A3)}$$

where $\bar{\mathscr{T}}_{12}$ and $\bar{\mathscr{R}}_{12}$ are local transmission and reflection operators for the single interface at $z = 0$. In the above, we have made use of the fact that all the operators are diagonal, and hence act like scalars and commute with each other. Solving for $\boldsymbol{A}_2$, we have

$$\boldsymbol{A}_2 = \left(\bar{\mathscr{T}} - \bar{\mathscr{R}}_{21} : \tilde{\bar{\mathscr{R}}}_{23} : e^{i2\mathscr{K}_{2z}h_2}\right)^{-1} : \bar{\mathscr{T}}_{12} : e^{i\mathscr{K}_{1z}z'} : \bar{\mathscr{G}}_1 : \tilde{\boldsymbol{J}}_{1s}. \quad \text{(A4)}$$

Since all the operators are diagonal, the inverse in (A4) is easily sought.

The upgoing wave in region 1 is a consequence of the reflection of the downgoing wave in region 1 plus a transmission of the upgoing wave in region 2. Consequently, we deduce that

$$\tilde{\bar{\mathscr{R}}}_{12} = \bar{\mathscr{R}}_{12} + \bar{\mathscr{T}}_{21} : e^{i2\mathscr{K}_{2z}h_2} : \tilde{\bar{\mathscr{R}}}_{23} : \left(\bar{\mathscr{I}} - \bar{\mathscr{R}}_{21} : \tilde{\bar{\mathscr{R}}}_{23} : e^{i2\mathscr{K}_{2z}h_2}\right)^{-1} : \bar{\mathscr{T}}_{12}. \quad \text{(A5)}$$

The above forms a recursive relationship between the generalized reflection operators $\tilde{\bar{\mathscr{R}}}_{12}$ and $\tilde{\bar{\mathscr{R}}}_{23}$. We can use this relationship recursively until we reach the bottommost layer, where the reflection operator is zero. Hence, the generalized reflection operator for each region can be found.

Also, from (A4), the amplitude of the downgoing wave in region i is related, in general, to the amplitude of the downgoing wave in region $i-1$ by the relationship,

$$\boldsymbol{A}_i = \left(\bar{\mathscr{I}} - \bar{\mathscr{R}}_{i,i-1} : \tilde{\bar{\mathscr{R}}}_{i,i+1} : e^{i2\mathscr{K}_{iz}h_i}\right)^{-1} : \bar{\mathscr{T}}_{i-1,i} : e^{i\mathscr{K}_{i-1,z}h_{i-1}} : \boldsymbol{A}_{i-1}. \quad \text{(A6)}$$

Equations (A5) and (A6) allow us to find the field amplitude everywhere given a source in region 1. Equation (A6), when used recursively, allows us to relate the downgoing wave in region j given that we know the downgoing wave in region i, where $i < j$ and regions i and j are both below the source. Hence, we can define a generalized transmission operator $\tilde{\bar{\mathscr{T}}}_{ij}$ which transmits a downgoing wave from region i to region j. From (A6), it is given by

$$\tilde{\bar{\mathscr{T}}}_{ij} = \prod_{k=i+1}^{j} \left(\bar{\mathscr{I}} - \bar{\mathscr{R}}_{k,k-1} : \tilde{\bar{\mathscr{R}}}_{k,k+1} : e^{i2\mathscr{K}_{kz}h_k}\right)^{-1} : \bar{\mathscr{T}}_{k-1,k} : e^{i\mathscr{K}_{k-1,z}h_{k-1}}. \quad \text{(A7)}$$

With this definition, $\boldsymbol{A}_j = \tilde{\bar{\mathscr{T}}}_{ij} : \boldsymbol{A}_i$. Note that if the layered medium had been above the source, we could still derive generalized reflection and transmission operators. The transmission operator in this case relates the upgoing wave in one region to the upgoing wave in another region.

We have derived the generalized reflection and transmission operators in terms of the local reflection and transmission operators. The local reflection operator $\bar{\mathscr{R}}_{ij}$ is a representation of the reflection matrix $\bar{\boldsymbol{R}}_{ij}$, which is related to the Fresnel reflection coefficients, i.e.,

$$\bar{\boldsymbol{R}}_{ij} = \begin{bmatrix} -R_{ij}^{\mathrm{TM}} & 0 \\ 0 & R_{ij}^{\mathrm{TE}} \end{bmatrix} \quad \text{(A8)}$$

where

$$R_{ij}^{\mathrm{TM}} = \frac{\epsilon_j k_{iz} - \epsilon_i k_{jz}}{\epsilon_j k_{iz} + \epsilon_i k_{jz}} \quad \text{and} \quad R_{ij}^{\mathrm{TE}} = \frac{\mu_j k_{iz} - \mu_i k_{jz}}{\mu_j k_{iz} + \mu_i k_{jz}}. \quad \text{(A8a)}$$

In the above $\bar{\mathscr{T}}_{ij} = \bar{\mathscr{I}} + \bar{\mathscr{R}}_{ij}$. The above derivation is similar to that of [27], but not identical.

B. Field Due to an Embedded Strip

When a strip is embedded in region j as shown in Fig. 4, we can write the field in region j as

$$\boldsymbol{E}_{js}(\boldsymbol{r}) = \bar{\boldsymbol{F}}(\boldsymbol{r}_s) : \Big(e^{i\mathcal{K}_{jz}|z-z_j|} + e^{i\mathcal{K}_{jz}(z-d_j)} : \bar{\mathcal{U}}_j : e^{i\mathcal{K}_{jz}(z_j-d_j)} + e^{-i\mathcal{K}_{jz}(z-d'_{j-1})} : \bar{\mathcal{D}}_j : e^{i\mathcal{K}_{jz}(d'_{j-1}-z_j)} \Big) : \bar{\mathcal{G}}_j : \tilde{\boldsymbol{J}}^I_{js}. \quad \text{(A9)}$$

$\bar{\mathcal{U}}_j$ and $\bar{\mathcal{D}}_j$ would be zero if there were no layered media above and below the strip. To find $\bar{\mathcal{U}}_j$ and $\bar{\mathcal{D}}_j$, we use the fact that at $z = d'_{j-1}$ the upgoing wave and the downgoing wave are related by the generalized reflection operator $\tilde{\bar{\mathcal{R}}}_{j,j-1}$, or

$$\bar{\mathcal{D}}_j : e^{i\mathcal{K}_{jz}(d'_{j-1}-z_j)} = \tilde{\bar{\mathcal{R}}}_{j,j-1} : \Big(e^{i\mathcal{K}_{jz}(d'_{j-1}-z_j)} + e^{i\mathcal{K}_{jz}h_j} : \bar{\mathcal{U}}_j : e^{i\mathcal{K}_{jz}(z_j-d_j)} \Big). \quad \text{(A10a)}$$

Similarly, at $z = d_j$, we have

$$\bar{\mathcal{U}}_j : e^{i\mathcal{K}_{jz}(z_j-d_j)} = \tilde{\bar{\mathcal{R}}}_{j,j+1} : \Big(e^{i\mathcal{K}_{jz}(z_j-d_j)} + e^{i\mathcal{K}_{jz}h_j} : \bar{\mathcal{D}}_j : e^{i\mathcal{K}_{jz}(d'_{j-1}-z_j)} \Big). \quad \text{(A10b)}$$

We can solve (A10a) and (A10b) for $\bar{\mathcal{U}}_j$ and $\bar{\mathcal{D}}_j$. Finally, we have

$$\bar{\mathcal{U}}_j : e^{i\mathcal{K}_{jz}(z_j-d_j)} = \Big(\bar{\mathcal{I}} - \tilde{\bar{\mathcal{R}}}_{j,j+1} : \tilde{\bar{\mathcal{R}}}_{j,j-1} : e^{i\mathcal{K}_{jz}2h_j} \Big)^{-1} : \tilde{\bar{\mathcal{R}}}_{j,j+1} : \Big(e^{i\mathcal{K}_{jz}(z_j-d_j)} + e^{i\mathcal{K}_{jz}h_j} : \tilde{\bar{\mathcal{R}}}_{j,j-1} : e^{i\mathcal{K}_{jz}(d'_{j-1}-z_j)} \Big) \quad \text{(A11a)}$$

$$\bar{\mathcal{D}}_j : e^{i\mathcal{K}_{jz}(d'_{j-1}-z_j)} = \Big(\bar{\mathcal{I}} - \tilde{\bar{\mathcal{R}}}_{j,j+1} : \tilde{\bar{\mathcal{R}}}_{j,j-1} : e^{i\mathcal{K}_{jz}2h_j} \Big)^{-1} : \tilde{\bar{\mathcal{R}}}_{j,j-1} : \Big(e^{i\mathcal{K}_{jz}(d'_{j-1}-z_j)} + e^{i\mathcal{K}_{jz}h_j} : \tilde{\bar{\mathcal{R}}}_{j,j+1} : e^{i\mathcal{K}_{jz}(z_j-d_j)} \Big). \quad \text{(A11b)}$$

Once $\bar{\mathcal{U}}_j$ and $\bar{\mathcal{D}}_j$ are found, the upgoing and downgoing waves in region j are known. Then we can find the generalized reflection and transmission operators in each region using the method described in the previous section. Using them, we can propagate the upgoing wave in region j upward, and the downgoing wave in region j downward and find the field everywhere.

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