Variable Preconditioning with Nested Iterations Employing MLFMA

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Iterative solutions of large electromagnetic scattering problems via the multilevel fast multipole algorithm (MLFMA) require strong preconditioners, especially if the formulation is based on the electric-field integral equation, which is the only valid formulation for targets involving open surfaces. MLFMA stores only the near-field part of the system matrix. For preconditioning purposes explicitly available sparse near-field matrix serves as a first-order approximation to the full system matrix. However, the exact solution of the near-field matrix is not efficient since it becomes prohibitively expensive due to the fill-ins. Instead, approximate solutions of the near-field matrix (such as, incomplete/approximate LU factors or sparse approximate inverses) are used to generate preconditioners that serve as a second level of approximation to the full system matrix.

Real-life applications require the solution of very large systems. However, as the number of unknowns grows, the near-field matrix becomes increasingly sparser. Then, even the exact solution of the near-field matrix may not yield low iteration counts. In that case, the preconditioner should be based on the solution of a matrix that is denser than the near-field matrix (e.g., the full system matrix or a slightly sparser version of it). A preconditioner based on the solution of such a matrix, whose entries are not explicitly available, can be constructed by using an iterative solver employing MLFMA-based matrix-vector multiplications. This variable preconditioning scheme requires a nested implementation of the iterative solvers, one for the solution of the original system matrix (i.e., outer iterations) and another for the solution of the preconditioner matrix (i.e., inner iterations).

In this study, we focus on using a Krylov subspace solver for the preconditioning operation, in order to accelerate the outer iterations and to achieve reduction in the overall computational time. For the outer iterations, the Krylov subspace methods that can be used with such a variable preconditioning scheme are called flexible Krylov solvers. Variable preconditioning with an inner solver using a low-accuracy MLFMA was shown to be successful in a previous work (G. Alléon et al., CERFACS Technical Report, TR/PA/03/65). The present study extends the previous work by constructing the preconditioner at various levels of approximations ranging from accurate solutions of the near-field matrix to the approximate solutions of the full system matrix. The reduction in the number of the outer iterations depends on two factors: how much the preconditioning system approximates the actual operator and how accurately we solve that system. We investigate how to balance these factors so that the overall computational and memory costs are minimized.