Preconditioned Iterative Solution of the Combined-field Integral Equation with the MLFMA

Levent Gürel, **Özgür Ergül**, **Tahir Malas**

Bilkent University, Turkey

Iterative solution of large problems of computational electromagnetics, which are obtained from mathematical formulations of real-life electromagnetic problems, is considered. In particular, our work on the fast multipole method (FMM) and the multi-level fast multipole algorithm (MLFMA), iterative solvers, preconditioners, and integral-equation formulations will be emphasized. Our efforts to reduce the number of iterations will be presented within the context of electric-field integral equation (EFIE), magnetic-field integral equation (MFIE), and combined-field integral equation (CFIE). The effects of various iterative solvers and preconditioners on the iteration counts will also be addressed.

Combining the EFIE with the MFIE to obtain the CFIE can also be interpreted as a mode of preconditioning. In this context, we consider the simultaneous solution of the EFIE and MFIE in the least-squares (LS) sense, instead of adding them into a single equation. Simultaneous solution of the square EFIE matrix equation and the square MFIE matrix equation requires the solution of a rectangular CFIE matrix equation. For this purpose, the MLFMA is employed within the framework of a stable LSQR algorithm by carefully implementing the matrix-vector products involving the Hermitian system, in addition to the regular matrix-vector products, to preserve the $O(N \log N)$ complexity.

In addition, explicit preconditioning schemes will be reported. Both exact and approximate inverses of sparse matrices containing electromagnetic near-field interactions of various strengths are used as preconditioners. Thresholding and reordering of the sparse near-field matrix are considered. Comparisons with some other popular preconditioners will be given. Spectra, convergence rates, and other relevant metrics of preconditioned linear systems derived from computational electromagnetics will be studied.