Hybrid CFIE-EFIE Solution of Composite Geometries with Coexisting Open and Closed Surfaces †

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Abstract

The combined-field integral equation (CFIE) is employed to formulate the electromagnetic scattering and radiation problems of composite geometries with coexisting open and closed conducting surfaces. Conventional formulations of these problems with the electric-field integral equation (EFIE) lead to inefficient solutions due to the ill-conditioning of the matrix equations and the internal-resonance problems. The hybrid CFIE-EFIE technique introduced in this paper, based on the application of the CFIE on the closed surfaces and EFIE on the open surfaces, significantly improves the efficiency of the solution.

I. Introduction

In this paper, we consider the solution of the scattering and radiation problems involving both open and closed conducting surfaces as shown in Fig. 1(a). These problems often arise in the simulations of practical electromagnetic scenarios, where the thin and thick conducing parts of the objects are modelled with open and closed surfaces, respectively. The formulation of these problems are customarily achieved by the electric-field integral equation (EFIE) due to the presence of the open surfaces. However, the EFIE is prone to internal-resonance problems and also leads to ill-conditioned matrix equations that decrease the performances of the iterative solvers, especially when the problem size is large.

For the electromagnetic modelling of problems involving closed conducting surfaces, the combined-field integral equation (CFIE) is preferred mainly because it is free of the internal resonance problems [1]. In addition, although this equation is simply the linear combination of the EFIE and the magnetic-field integral equation (MFIE), i.e.,

$$\text{CFIE} = \alpha\text{EFIE} + (1-\alpha)\text{MFIE}, \tag{1}$$

it generates considerably better-conditioned matrix equations compared to both the EFIE and MFIE [2]. This crucial property of the CFIE becomes useless for the problems of open surfaces, where the EFIE becomes the inevitable choice. In consolation, the EFIE is generally observed to be performing better for problems of solely open surfaces, having rapid convergence of the iterative solutions, especially when preconditioned properly.

In this paper, we investigate the solution of the composite-geometry problems involving both open and closed conducting surfaces, where the EFIE solutions become inefficient due to the presence of the closed parts. We improve the solution of these problems by

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taking a hint from the improvement by the CFIE in the solution of the scattering problems involving only closed surfaces. The proposed technique is based of the application of a hybrid CFIE-EFIE formulation, leading to better-conditioned matrix equations and consequently more efficient iterative solutions. In the implementations using the hybrid technique, the open parts of the problems are still formulated with the EFIE, while the CFIE is enforced on the closed parts to improve the conditioning. An example involving a radiation problem will be given to demonstrate the overall improvement by the hybrid formulation compared to the use of the pure EFIE on the same problem.

II. Hybrid CFIE-EFIE Formulation

For conducting surfaces, numerical application of the EFIE and MFIE leads to matrix equations as

$$\sum_{n=1}^{N} Z_{mn}^{E,M} a_n = v_m^{E,M}, \qquad m = 1, ..., N, \tag{2}$$

where the matrix elements, namely the interactions between the basis functions $\boldsymbol{b}_n(\boldsymbol{r})$ and testing functions $\boldsymbol{t}_m(\boldsymbol{r})$, are derived as

$$\begin{aligned} Z_{mn}^{E} = & \int_{S_m} d\boldsymbol{r} \boldsymbol{t}_m(\boldsymbol{r}) \cdot \int_{S_n} d\boldsymbol{r}' \boldsymbol{b}_n(\boldsymbol{r}') g(\boldsymbol{r}, \boldsymbol{r}') \\ & - \frac{1}{k^2} \int_{S_m} d\boldsymbol{r} \boldsymbol{t}_m(\boldsymbol{r}) \cdot \int_{S_n} d\boldsymbol{r}' \boldsymbol{b}_n(\boldsymbol{r}') \cdot \left[\nabla \nabla' g(\boldsymbol{r}, \boldsymbol{r}') \right] \end{aligned} \tag{3}$$

for the EFIE, and as

$$Z_{mn}^{M} = \int_{S_m} d\boldsymbol{r} \boldsymbol{t}_m(\boldsymbol{r}) \cdot \boldsymbol{b}_n(\boldsymbol{r}) - \int_{S_m} d\boldsymbol{r} \boldsymbol{t}_m(\boldsymbol{r}) \cdot \hat{\boldsymbol{n}} \times \int_{S_n} d\boldsymbol{r}' \boldsymbol{b}_n(\boldsymbol{r}') \times \nabla' g(\boldsymbol{r}, \boldsymbol{r}') \tag{4}$$

for the MFIE. To form the CFIE system, the matrix elements in Eqs. (3) and (4) are linearly combined as

$$Z_{mn}^{C} = \alpha_m Z_{mn}^{E} + (1 - \alpha_m) \frac{i}{k} Z_{mn}^{M}. \tag{5}$$

Different from (1), where α is a constant, the parameter α_m to weight the EFIE and MFIE contributions in the CFIE is flexible in (5). Such a definition provides the freedom of choosing different linear combinations for different testing functions. Consequently, it becomes possible to employ the CFIE with $\alpha_m \neq 0$ for the testing functions located on the closed parts of the geometry while setting $\alpha_m = 1$ to use the EFIE on the open parts. As a result, by the inclusion of the CFIE interactions into the matrix equation, we obtain better-conditioned formulations leading to faster converging iterative solutions compared to the pure EFIE.

III. Results

To demonstrate the improvement by the use of the hybrid CFIE-EFIE formulation, we present the results of a radiation problem involving a dipole antenna placed over a perfectly conducting rectangular box with dimensions $\lambda \times 5\lambda \times 5\lambda$ as depicted in Fig. 1(a). The dipole antenna is modelled by a strip of zero thickness with length λ and width $\lambda/10$. The composite structure is triangulated with a mesh size of $\lambda/10$ corresponding to 16,124 triangles on the box and only 20 triangles on the strip. The feed of the antenna is simulated by a delta-gap source located at the center of the strip as shown in Fig. 1(b).

Local electric field with amplitude $1/d$ is defined inside the infinitely narrow opening between the triangles so that

$$v_m^C = \alpha_m v_m^E + (1-\alpha_m) v_m^M = v_m^E$$
$$= \lim_{d\to 0} \frac{i}{k\eta} \int_{S_m} d\boldsymbol{r} \boldsymbol{t}_m(\boldsymbol{r}) \cdot \boldsymbol{E}^i(\boldsymbol{r}) = \frac{il_m}{k\eta} \delta[m,e], \quad \delta[m,e] = \left\{ \begin{array}{ll} 1 & m=e \\ 0 & m \neq e \end{array} \right\}. \quad (6)$$

Consequently, only a single element of the excitation vector v_m^C, i.e., $m = e$ will be nonzero. The resulting radiation problem is solved with the multi-level fast multipole algorithm [3] employing the Rao-Wilton-Glisson [4] functions as the basis and testing functions defined on the triangular domains, leading to a linear system with $24,205$ unknowns.

Fig. 2(a) demonstrates the iteration counts for the solution of the problem with the conjugate gradient squared (CGS) algorithm with respect to the value of α_m in (5) applied on the closed parts of the geometry, i.e., on the rectangular box. The dashed and solid curves represent the number of iterations required to reach 10^{-3} and 10^{-6} residual errors, respectively, while employing a block-diagonal preconditioner (BDP) with 561,221 nonzero elements to accelerate the iterative solution. Both curves are observed to be minimized when α_m is about 0.2–0.3, with significant improvement in the convergence compared to the pure EFIE solution of the problem ($\alpha_m = 1 \quad \forall m$), which is not shown in the figure due to the extremely high iteration counts.

To obtain convergent solutions with the EFIE formulation, strong preconditioners are required as presented in Fig. 2(b), where the iterative solver does not converge with a BDP and thus needs to be accelerated with a near-field preconditioner (NFP) with 5,267,535 nonzero elements obtained by keeping all of the near-field interactions in the impedance matrix. However, even with this strong preconditioner that requires extensive use of the memory and leads to significant increase the processing time, the residual error does not drop under 10^{-6} until the 1400th iteration. The improvement by the hybrid formulation is demonstrated in Fig. 3(a), where the convergence characteristics of the EFIE, the hybrid MFIE-EFIE ($\alpha_m = 0$ on the closed part), and the hybrid CFIE-EFIE ($\alpha_m = 0.2$ on the closed part) formulations are depicted on the same graph. With the same BD preconditioner, the convergence is significantly improved by the use of the CFIE on the closed part of the geometry. Finally, Fig. 3(b) shows the normalized radar cross section (RCS/λ^2 in dB) values on the z-x plane with respect to θ, where we observe the accuracy of the hybrid CFIE-EFIE compared to the reference EFIE solution.

IV. Conclusion

A novel hybrid CFIE-EFIE formulation is presented for the solution of the composite-geometry problems with coexisting open and closed conducting surfaces. With the application of the technique to the radiation and scattering problems involving composite geometries, the efficiency of the solutions can be significantly improved compared to using only the EFIE formulation on the whole geometry.

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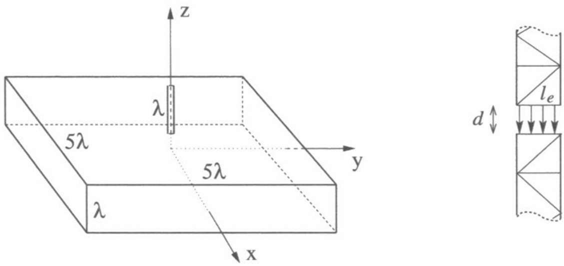
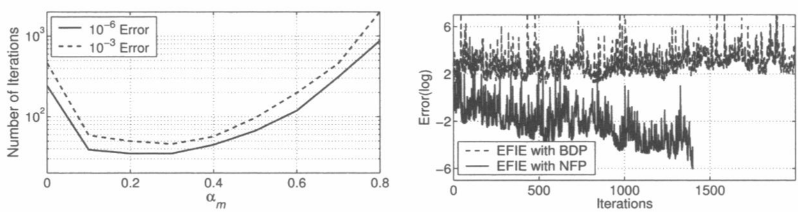


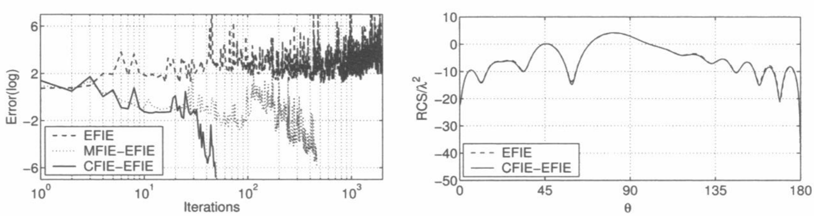
(a) (b)

Fig. 1. (a) Composite geometry involving a dipole antenna (open surface) of length λ located over a perfectly conducting rectangular box (closed surface) with the dimensions of $\lambda \times 5\lambda \times 5\lambda$. (b) Delta-gap source located at the center of the antenna.



(a) (b)

Fig. 2. (a) Iteration counts for the solution of the radiation problem in Fig. 1(a) with the CGS algorithm with respect to the value of α_m. (b) Convergence characteristics of the EFIE formulation using the BDP and NFP to accelerate the iterative solution.



(a) (b)

Fig. 3. (a) Comparison of convergence characteristics of the EFIE, the MFIE-EFIE, and the CFIE-EFIE formulations using BDP. (b) Normalized radar cross section (RCS/λ^2 in dB) on the z-x plane with respect to θ.