Iterative Least-Squares Solution of the Combined-Field Integral Equation[†]

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We consider the iterative solution of large problems of computational electromagnetics using the fast multipole method (FMM) and the multi-level fast multipole algorithm (MLFMA). Even though these fast solvers are quite successful to perform the matrix-vector products efficiently and accurately at each iteration, it is desirable to reduce the number of iterations as much as possible, not only by improving the conditioning of the resulting linear systems with appropriate preconditioning techniques, but also by choosing the core iterative algorithm properly with respect to the nature of the problem and its formulation.

Combining the electric-field integral equation (EFIE) with the magnetic-field integral equation (MFIE) to obtain the combined-field integral equation (CFIE) can also be interpreted as a mode of preconditioning. In this talk, we consider the simultaneous solution of the EFIE and MFIE in the least-squares (LS) sense, instead of adding them into a single equation. Simultaneous solution of the square EFIE matrix equation and the square MFIE matrix equation requires the solution of a rectangular CFIE matrix equation. For this purpose, the MLFMA is employed within the framework of a stable LSQR algorithm (*C. C. Paige and M. A. Saunders, ACM Trans. Math. Software, 8, 43–71, 1982*) by carefully implementing the matrix-vector products involving the Hermitian system, in addition to the regular matrix-vector products, to preserve the $O(N \log N)$ complexity.

The LSQR algorithm, which is based on the bidiagonalization of the linear system, will be shown to exhibit fast convergence, however, the overall residual of the rectangular system does not drop beyond a certain value. This is because the EFIE and the MFIE do not have exactly the same solutions, i.e., they are not compatible.

Solutions of the square EFIE, square MFIE, and square CFIE systems with the LSQR algorithm are also experimented. For the LS solution of square systems, it is observed that the exact residual drops monotonically since the system is compatible. Investigation of the convergence characteristics demonstrates that the LSQR algorithm holds promise to reduce the number of iterations and become preferable especially for the solution of weakly-conditioned problems.

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