Fast Computation of Scattering from (Near-)Resonant Structures

Levent Gürel* and Ali E. Yilmaz
Dept. of Electrical & Electr. Eng.
Bilkent University
Bilkent, Ankara, Turkey
(lgurel@ee.bilkent.edu.tr)

Weng Cho Chew
Ctr. for Computational Electromagnetics
Dept. of Electrical & Computer Eng.
University of Illinois, Urbana, IL 61801
(w-chew@uiuc.edu)

1 Introduction

A family of fast direct (noniterative) solvers, which could be classified as

- recursive solver for the scattered field (RSSF),
- recursive solver to invert the (impedance) matrix (RSIM), and
- recursive solver for the unknown (coefficient) vector (RSUV),

were outlined in [1]. These solvers are related to the recursive aggregate interaction matrix algorithm (RAIMA) [2], which is based on the recursive interaction matrix algorithm (RIMA) [3,4] and the recursive aggregate T-matrix algorithm (RATMA) [5,6]. The noniterative nature of the solvers used in this paper is emphasized due to the inability of iterative solvers to handle problems that involve resonant or near-resonant structures. Even the immensely successful fast iterative solvers [7–13] that have been developed in recent years share this drawback.

One example of (near-)resonant structures is a stack of closely spaced conducting patches, as depicted in Fig. 1. In order to illustrate that iterative solvers

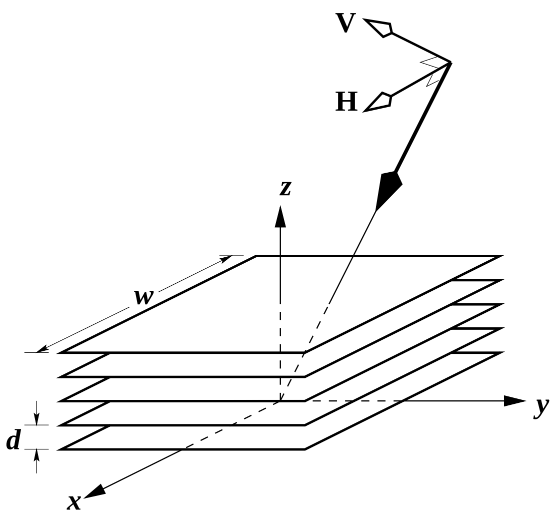

Fig. 1. Stack of closely spaced conducting patches.

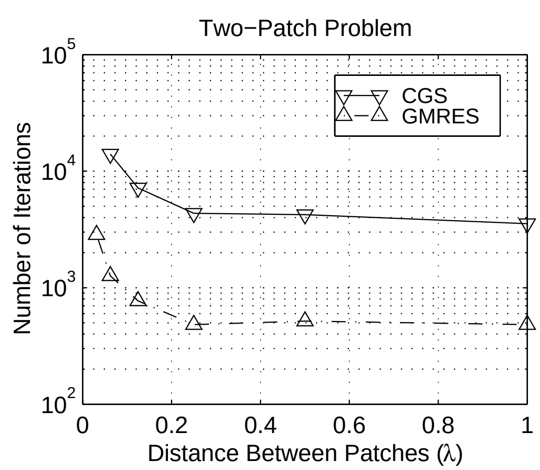

Fig. 2. Number of iterations as a function of spacing (d).

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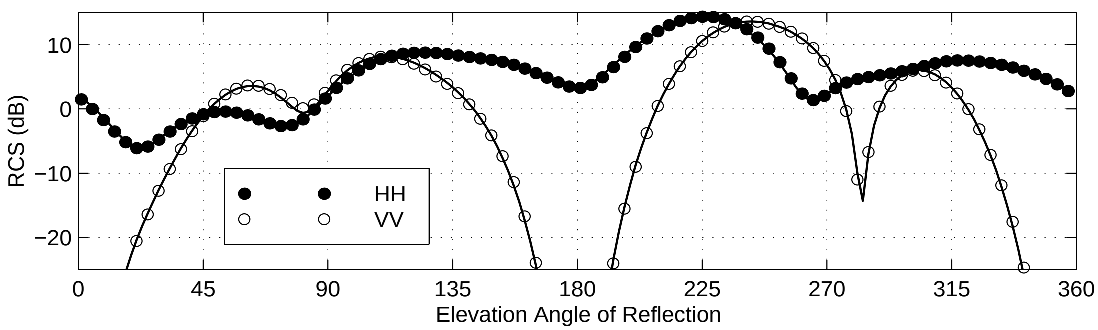
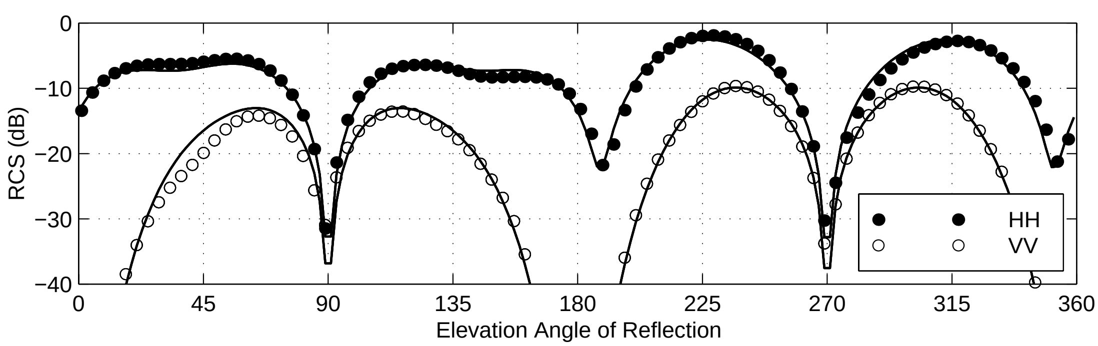
become useless for such problems, Fig. 2 shows how the number of iterations increase as d is reduced from λ to $\lambda/2$ for two $\lambda \times \lambda$ patches ($w = \lambda$). Direct solvers, such as Gaussian elimination and RIMA, are less sensitive to resonance problems, but require $O(N^3)$ operations to solve an N-unknown problem. The fast direct solvers utilized in this paper have $O(N^{7/3})$ or less complexity, which is significantly lower than $O(N^3)$.

2 Fast Direct Solvers

The direct solvers of this paper are recursive in nature, i.e., they introduce (sub)scatterers to the geometry one at a time and obtain the *complete* solution for a *partial* geometry at each recursion. As a new (sub)scatterer is added, its interaction with all the existing ones need to be computed. The existing (sub)scatterers are divided into two groups: near neighbors and distant neighbors with respect to the most recently added (sub)scatterer. The near-neighbor interactions are computed using RIMA [3,4], whereas the distant-neighbors are aggregated and their wholesale interaction with the recently added (sub)scatterer is efficiently computed using RATMA [5,6]. Detailed mathematical derivations of the solution algorithms are given in [14].



(a)



(b)

Fig. 3. RCS of the 16-patch stack computed with both RSSF (symbols) and MoM (solid curves): (a) $\phi = 0$ and (b) $\phi = \pi/2$ cuts.

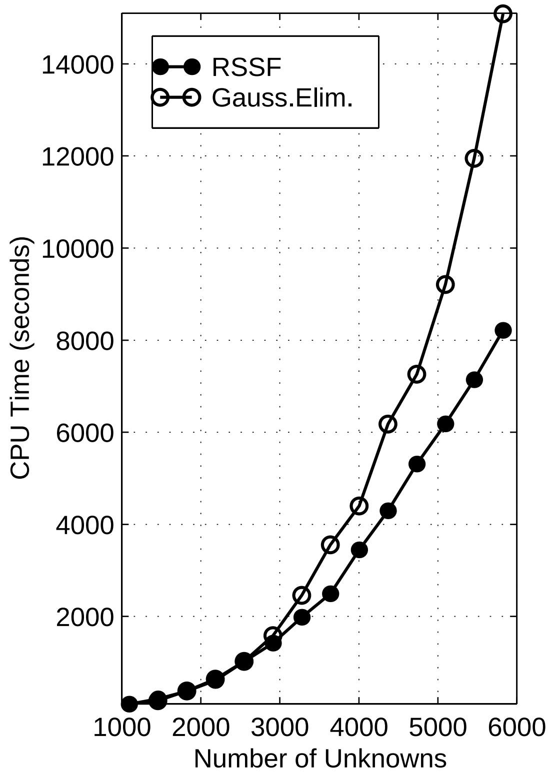


Fig. 4. Comparison of the CPU times required by RSSF and Gaussian elimination.

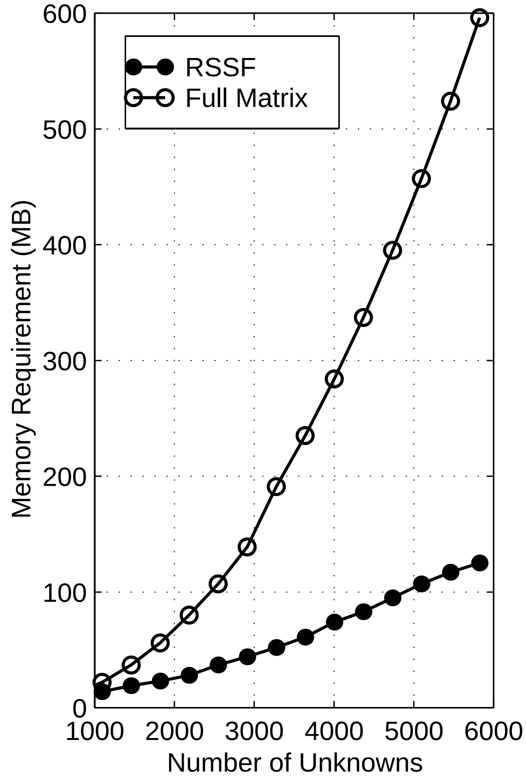


Fig. 5. Comparison of the memory requirements of RSSF and MoM employing a full matrix.

3 Performance Demonstrations

Three aspects of the performance of the direct solvers are demonstrated here: (i) accuracy, (ii) reduced computational complexity, and (iii) reduced memory requirement. Scattering from a stack of patches, as shown in Fig. 1, is used as an application problem, although the applicabilities of the solvers are not limited to this class of geometries. Consider the case of 16 patches with $w = \lambda$ and $d = \lambda/8$. Plane waves, whose electric fields are polarized in the horizontal (H) and vertical (V) directions (i.e., ϕ and θ polarizations, respectively) and have unit amplitudes, are incident on the structure in the direction of $(\theta, \phi) = (225°, 0°)$. Figure 3 shows the HH and VV polarizations of the RCS computed with both one of the fast direct solvers of this paper (RSSF) and the method of moments (MoM) on the $\phi = 0$ and $\phi = \pi/2$ cuts. The agreement between the two sets of solutions testifies to the accuracy of fast direct solver RSSF.

Figure 4 shows a comparison of the CPU times required by RSSF and Gaussian elimination as the number of patches are increased from 4 to 16. Finally, Fig. 5 shows a comparison of the memory requirements of the two direct solution algorithms. Clearly, both the CPU-time and the memory requirements of RSSF are lower than those of MoM employing a full matrix and Gaussian elimination.

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4 Other Applications

Besides being alternatives to iterative solvers, fast direct solvers can also be used in the framework of iterative solvers as preconditioners. Among the three direct solvers mentioned in this paper, the recursive solver that computes the unknown coefficient vector (RSUV) is especially suited for this task. In the case of block-diagonal preconditioners, for instance, the reduced complexity of the fast direct solvers would allow for the solution of blocks with larger sizes, resulting in better preconditioning and thus faster convergence.

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