EFFICIENT METHODS FOR ELECTROMAGNETIC CHARACTERIZATION OF 2-D GEOMETRIES IN STRATIFIED MEDIA

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I. Introduction

Numerically efficient method of moments (MoM) algorithms are developed for and applied to 2-D geometries in multilayer media. These are, namely, the spatial-domain MoM in conjunction with the closed-form Green's functions [1], the spectral-domain MoM using the generalized pencil of functions (GPOF) algorithm [2] and FFT algorithm to evaluate the MoM matrix entries. These approaches are mainly to improve the computational efficiency of the evaluation of the MoM matrix entries. Among these, the spectral-domain MoM using the GPOF algorithm is the most efficient approach for printed multilayer geometries. The assessment of the efficiency of this method is performed on several problems, by comparing the matrix fill times for these three approaches.

II. Formulation

The first step of the MoM formulation is to write an integral equation describing the electromagnetic problem, which could be the mixed potential integral equation (MPIE) or the electric field integral equation (EFIE) for the printed geometries. These integral equations require related Green's functions, either of the vector and scalar potentials (for MPIE formulation) or of the electric fields (for EFIE formulation). Green's functions of the vector and scalar potentials in the spectral and spatial domains are obtained for the sources of horizontal and vertical electric dipoles placed in multilayer planar media, where the layers are assumed to extend to infinity in transverse directions [3].

The scattered electric fields for TE and TM excitations can be written for a planar geometry (printed on $x-y$ plane) as

$$E_x^s = -j\omega G_{xx}^A * J_x + \frac{1}{j\omega}\frac{\partial}{\partial x}\left(G_x^{q_e} * \frac{\partial}{\partial x} J_x\right) \tag{1}$$

$$E_y^s = -j\omega G_{yy}^A * J_y \tag{2}$$

and their Fourier transforms are

$$\tilde{E}_x^s = -j\omega\left(\tilde{G}_{xx}^A - \frac{k_x^2}{\omega^2}\tilde{G}_x^{q_e}\right)\tilde{J}_x(k_x) = -j\omega\tilde{G}_{xx}^E\tilde{J}_x(k_x) \tag{3}$$

$$\tilde{E}_y^s = -j\omega\tilde{G}_{yy}^A\tilde{J}_y(k_x) \tag{4}$$

For the calculation of the incident field, a simple case in 2-D represented in Figure 1 is considered. Since the strips are horizontal, we only need the incident field in

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x-direction for TE excitation and in y-direction for TM excitation,

$$E_x^i = -\frac{k\cos\theta}{\omega\epsilon} e^{jk\sin\theta x}\left(e^{jk\cos\theta z} - \tilde{R}_{TM}^{i,i-1} e^{jk\cos\theta z}\right) \tag{5}$$

$$E_y^i = e^{jk\sin\theta x}\left(e^{jk\cos\theta z} + \tilde{R}_{TE}^{i,i-1} e^{jk\cos\theta z}\right) \tag{6}$$

where $k_z = k\cos\theta$ and $k_x = k\sin\theta$. Since the total electric fields are the summation of the scattered and incident electric fields, the boundary conditions for the tangential electric fields for both TM and TE excitations are applied on the conducting body.

The basis and testing functions are chosen as triangular functions for the TE excitation :

$$B_x(x) = \begin{cases} \frac{x - x_1}{h_x} & \text{if } x_1 \le x \le x_2 \\ \frac{x_3 - x}{h_x} & \text{if } x_2 \le x \le x_3 \\ 0 & \text{otherwise} \end{cases} \quad ; \quad \tilde{B}_x(k_x) = h_x e^{jk_x x_2} sinc^2\left(\frac{k_x h_x}{2}\right) \tag{7}$$

and, pulse functions for the TM excitation :

$$B_y(x) = \begin{cases} 1 & \text{if } x_1 \le x \le x_3 \\ 0 & \text{otherwise} \end{cases} \quad ; \quad \tilde{B}_y(k_x) = h_x e^{jk_x x_2} sinc\left(\frac{k_x h_x}{2}\right) \tag{8}$$

where $x_2 - x_1 = x_3 - x_2 = h_x$. Consequently, the application of the boundary condition on the total electric fields via the testing procedure results in the following set of linear equations: For TE excitation

$$-jw \sum_{m=1}^{M_{te}} I_{xm} \left\langle \tilde{T}_{xn}^* \, , \, \tilde{G}_{xx}^E \tilde{B}_{xm} \right\rangle = - < \tilde{T}_{xn}^*, \tilde{E}_x^i > \quad \text{for} \;\; n = 1, ..., M_{te} \tag{9}$$

and each term of the impedance matrix can be written explicitly as

$$\left\langle \tilde{T}_{xn}^* \, , \, \tilde{G}_{xx}^E \tilde{B}_{xm} \right\rangle = \int_{-\infty}^{\infty} dk_x e^{-jk_x(x_{n_2} - x_{m_2})} h_x^2 sinc^4\left(\frac{k_x h_x}{2}\right) \tilde{G}_{xx}^E \tag{10}$$

For TM excitation

$$-jw \sum_{m=1}^{M_{tm}} I_{ym} \left\langle \tilde{T}_{yn}^* \, , \, \tilde{G}_{yy}^A \tilde{B}_{ym} \right\rangle = - < \tilde{T}_{yn}^*, \tilde{E}_y^i > \quad \text{for} \;\; n = 1, ..., M_{tm} \tag{11}$$

and each term of the impedance matrix can be written explicitly as

$$\left\langle \tilde{T}_{yn}^* \, , \, \tilde{G}_{yy}^A \tilde{B}_{ym} \right\rangle = \int_{-\infty}^{\infty} dk_x e^{-jk_x(x_{n_2} - x_{m_2})} h_x^2 sinc^2\left(\frac{k_x h_x}{2}\right) \tilde{G}_{yy}^A \tag{12}$$

If Eqs. (10) and (12) are examined, it is observed that the exponential terms can be considered as the kernel of the Fourier transformation and the rest as the function to be transformed. Therefore, the matrix entries can be calculated using a FFT algorithm. Another approach to find the MoM matrix entries is that the whole integrand, except the kernel, are approximated in terms of complex exponentials using the GPOF algorithm. Hence, the matrix entries are evaluated analytically using Hankel identity [4] and each term of the impedance matrix for TE excitation can be given as

$$
\begin{aligned}
\left\langle \tilde{T}^{*}_{xn} \,,\, \tilde{G}^{E}_{xx}\tilde{B}_{xm} \right\rangle \;=\; & -\frac{j\pi h_x^2}{2}\left\{ \sum_{p_{0xx}=1}^{P_{0xx}} C_{p_{0xx}} H_0^{(2)}(k_i\,\rho_{p_{0xx}}) + \sum_{p_{1xx}=1}^{P_{1xx}} C_{p_{1xx}} H_0^{(2)}(k_i\,\rho_{p_{1xx}}) \right. \\
& + \sum_{p_{2xx}=1}^{P_{2xx}} C_{p_{2xx}} H_0^{(2)}(k_i\,\rho^{(1)}_{p_{2xx}}) + \sum_{p_{3xx}=1}^{P_{3xx}} C_{p_{3xx}} H_0^{(2)}(k_i\,\rho_{p_{3xx}}) \\
& \left. + \sum_{p_{2xx}=1}^{P_{2xx}} C_{p_{2xx}} H_0^{(2)}(k_i\,\rho^{(2)}_{p_{2xx}}) \right\} \qquad (13)
\end{aligned}
$$

where $C_{p_{ixx}}$ and $\rho_{p_{ixx}}$ for $i = 0, 1, 2, 3$ are obtained from the GPOF method.

III. Numerical Results

The geometry of the example is given in Figure 1, where Region 0 is PEC, $\epsilon_{r_1} = 4$ and $\epsilon_{r_2} = 1$, the width of the strips $2w = 0.5\lambda_2$, and $h_1 = h_2 = \lambda_2$. The number of the basis functions is chosen to be 110 for the TE case and 114 for the TM case, and the angle of incidence $\theta = 0^o$. Table 1 shows the CPU times (on a SUN SPARCstation 20/50) of all methods for the TE and TM excitations and, it is obvious that the spectral-domain approach is the most efficient one. Figure 2 shows the magnitudes of the current densities, obtained by using these three approaches, on the strips for the TE and TM excitations, respectively, no distinguishable difference in the results are observed.

IV. Conclusion

The application of the MoM to 2-D planar multilayer geometries transforms integral equations into matrix equations whose entries become double integrals over finite domains in the spatial-domain MoM, and single integrals over infinite domain in the spectral-domain MoM. In this work, three different algorithms to efficiently evaluate these integrals have been studied. It is observed that there is no accuracy problem in any of these approaches, but as far as the numerical efficiency of these algorithms are concerned, the one using the GPOF formulation in the spectral-domain MoM formulation is the best, which has been verified for several examples by giving the CPU times for filling-up the MoM matrices.

References

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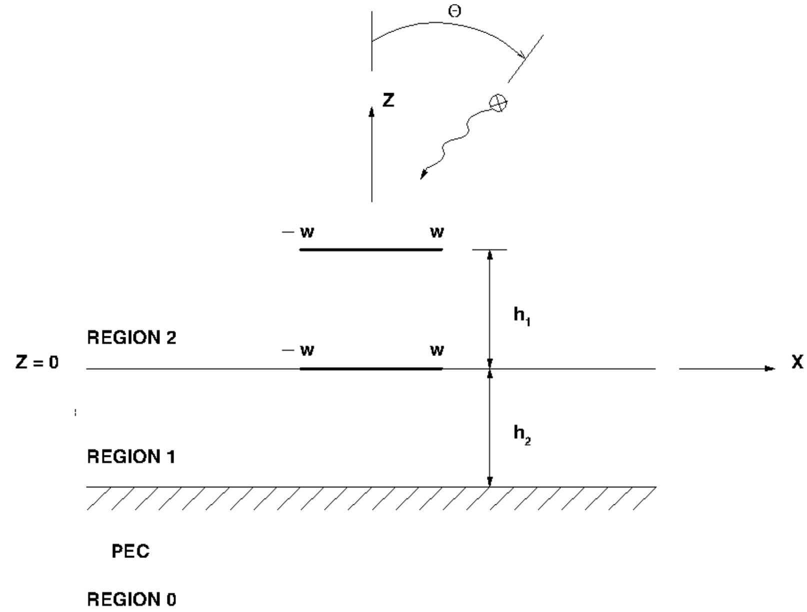

Figure 1: A two-strip, three layer geometry.

	CPU time (s)	
	TE	TM
Spectral	109.2	63.6
FFT	142.2	166.5
Spatial	6980.2	2187.1

Table 1: CPU times of the spectral domain, spatial domain and FFT approaches for TE and TM excitations

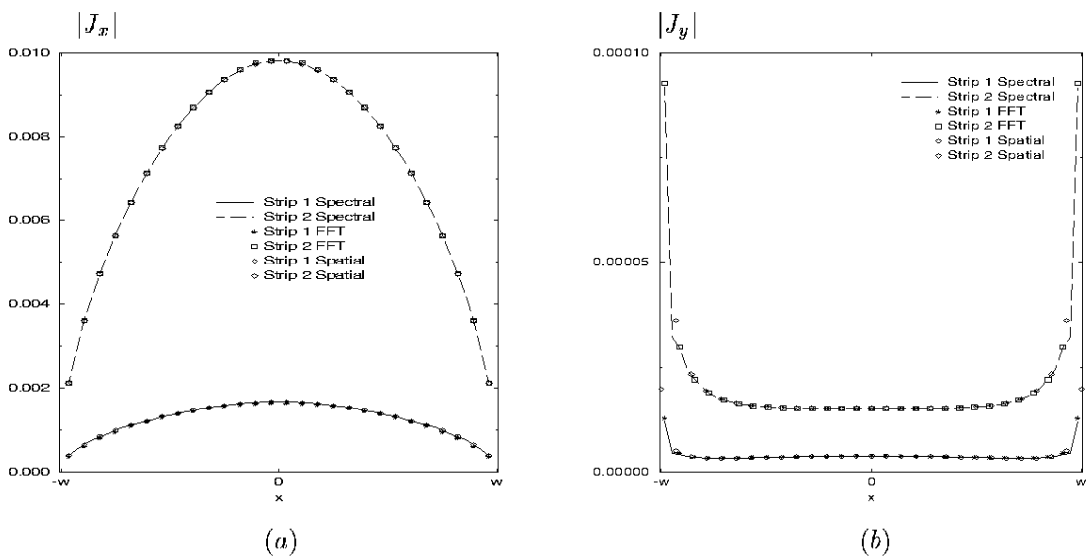

Figure 2: Magnitudes of the current densities on the two strips for a) TE excitation b) TM excitation