

Solutions of Extremely Large Electromagnetics Problems Involving Tens of Millions of Unknowns Using Parallel MLFMA and Preconditioners

Levent Gürel^{1,2}, Özgür Ergül^{1,2}, Tahir Malas^{1,2}

¹Department of Electrical and Electronics Engineering

²Computational Electromagnetics Research Center (BiLCEM)

Bilkent University, TR-06800, Bilkent, Ankara, Turkey

E-mail: lgurel@bilkent.edu.tr, ergul@ee.bilkent.edu.tr, tmalas@ee.bilkent.edu.tr

Abstract

We present fast and accurate solutions of very large electromagnetics problems discretized with tens of millions of unknowns. Scattering and radiation problems involving three-dimensional large-scale objects are formulated accurately with surface integral equations. The resulting dense matrix equations are solved iteratively by using a parallel implementation of the multilevel fast multipole algorithm (MLFMA). For efficient solutions, iterations are accelerated by employing a robust preconditioning technique based on a flexible iterative algorithm and an approximate MLFMA. We demonstrate the effectiveness of our solver on various scattering problems, including a sphere of radius 150λ discretized with 85,148,160 unknowns.

1. Surface Formulations of Problems in Electromagnetics

Surface integral equations are commonly used to formulate scattering and radiation problems in electromagnetics. For a perfectly conducting object, application of the boundary conditions on the surface leads to the electric-field integral equation (EFIE), the magnetic-field integral equation (MFIE), and the combined-field integral equation (CFIE), which is a convex combination of EFIE and MFIE. For the solution of problems involving closed conductors, CFIE is more preferable than EFIE and MFIE, because it is free of the internal-resonance problem, and it produces better-conditioned matrix equations. Numerical solutions of integral equations require the discretization of the objects and the expansion of the surface currents in a series of basis functions. Then, the integral equations are tested with a set of testing functions to derive $N \times N$ dense matrix equations. Solutions of these equations provide the expansion coefficients for the induced currents, which can be used to compute the scattered (or radiated) electric and magnetic fields.

For accurate solutions of integral equations, discretization elements should be small, compared to wavelength. Consequently, discretizations of objects with dimensions of several wavelengths lead to matrix equations with thousands of unknowns. These matrix equations can be solved iteratively, but the problem size is limited, due to high computational complexity. The multilevel fast multipole algorithm (MLFMA) reduces the complexity of the matrix-vector multiplications required by the iterative solvers from $O(N^2)$ to $O(N \log N)$, allowing for the solution of large problems with limited computational resources [1]. Using MLFMA, it becomes possible to solve problems with hundreds of thousands of unknowns on relatively inexpensive computing platforms. On the other hand, accurate solutions of many real-life problems require discretizations with millions of elements, which result in matrix equations with millions of unknowns. Due to the fact that these problems cannot easily be handled with the sequential implementations of MLFMA, it is helpful to increase computational resources by assembling parallel computing platforms and, at the same time, by parallelizing MLFMA.

2. Parallel Implementation of MLFMA

MLFMA calculates the far-field interactions (matrix elements) in a group-by-group manner. The groups are created by placing the object in a cubic box and recursively dividing the computational domain into subboxes. A multilevel tree structure is constructed by considering the boxes (clusters) at different levels. Then, the far-field interactions are calculated via three main stages, i.e., aggregation, translation, and disaggregation. The aggregation stage involves the calculation of the radiation patterns of the clusters from the lowest level to the top of the tree structure. Then, the radiation patterns are converted into incoming fields during the translation stage. Finally, the disaggregation stage is performed by computing the total incoming field for each cluster from the highest level to the bottom of the tree, where the incoming fields are distributed onto the testing functions. There are also $O(N)$ near-field interactions in MLFMA, which are calculated directly and stored in the memory.

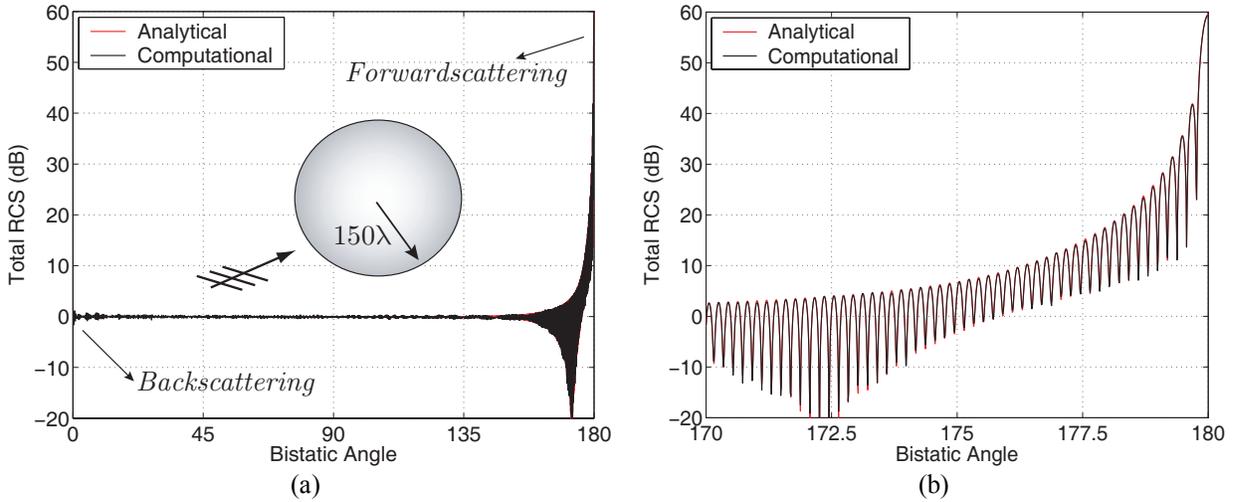


Figure 1. Bistatic RCS (in dB) of a sphere of radius 150λ discretized with 85,148,160 unknowns (a) from 0° to 180° and (b) from 170° to 180° , where 180° corresponds to the forward-scattering direction.

Parallelization of MLFMA is not trivial, due to its complicated tree structure. Simple parallelization strategies, based on distributing the clusters among processors, lead to inefficient solutions due to communications between the processors, insufficient load-balance of the workload among the processors, and unavoidable duplication of some of the computations over multiple processors. Advanced parallelization techniques based on distributing the radiated and incoming fields for the higher levels of the tree structure improve the efficiency significantly. Recently, we have developed a hierarchical parallelization strategy that involves the best partitioning of each level separately, by considering the simultaneous distribution of clusters and their fields [2]. This strategy presents superior performance compared to previous parallelization strategies and enables the efficient solution of very large problems on distributed-memory architectures.

As an example to the solution of very large problems, Fig. 1 presents the results of a scattering problem involving a perfectly conducting sphere of radius 150λ discretized with 85,148,160 unknowns. At this time, this is the solution of the largest integral-equation problem ever solved. The scattering problem is formulated with CFIE and solved with two digits of accuracy using a 9-level MLFMA parallelized into 16 Intel Xeon processors. The overall time involved in the setup and in the 23 BiCGStab iterations (for 0.001 residual error) is 710 minutes. The total memory, including that used for the near-field interactions, the radiation and receiving patterns of the basis and testing functions, the translation operators, and the radiated/incoming fields of the clusters, is 387 gigabytes. Fig. 1(a) depicts the normalized bistatic radar cross section (RCS/ πa^2 , where a is the radius of the sphere in meters) values in decibels (dB) from 0° to 180° , where 0° and 180° correspond to backscattering and forward-scattering directions, respectively. For a detailed comparison, Fig. 1(b) presents the same results from 170° to 180° . We observe that the computational values are in agreement with the analytical values obtained by a Mie-series solution.

Iterative solutions of CFIE problems involving simple geometries, such as spheres, are relatively easy to achieve with small numbers of iterations. For complicated objects, however, it is difficult to keep the iteration counts low, even when using CFIE. For the efficient solution of these problems, convergence of the iterations should be accelerated by employing robust preconditioning techniques.

3. Solution of Complicated Problems with Robust Preconditioning Techniques

As the problem size grows, iterative solutions of problems with MLFMA can be challenging. First of all, the condition numbers of the matrices grow rapidly with the increasing number of unknowns. Due to large condition numbers, efficient solutions with small numbers of iterations need denser preconditioners, which can be computationally expensive. In addition, only the near-field interactions are available in MLFMA to construct preconditioners; whereas these interactions may not provide a good approximation to the full matrix, especially when the problem size is large and the sparsity of the near-field interactions is small. As a consequence, for the efficient solution of large problems with MLFMA, we have developed a robust preconditioning technique based on a flexible GMRES (FGMRES) method and an approximate MLFMA (AMLFMA) [3]. Using a flexible solver, the

preconditioner is allowed to change during the iterations. As the preconditioner, we perform the iterative solution of an approximate matrix equation, which is obtained by relaxing the accuracy of MLFMA in a controllable manner. This way, the resulting AMLFMA preconditioner effectively uses the far-field interactions, in addition to the near-field interactions. The inner solutions are further accelerated by simple preconditioners based on the near-field interactions.

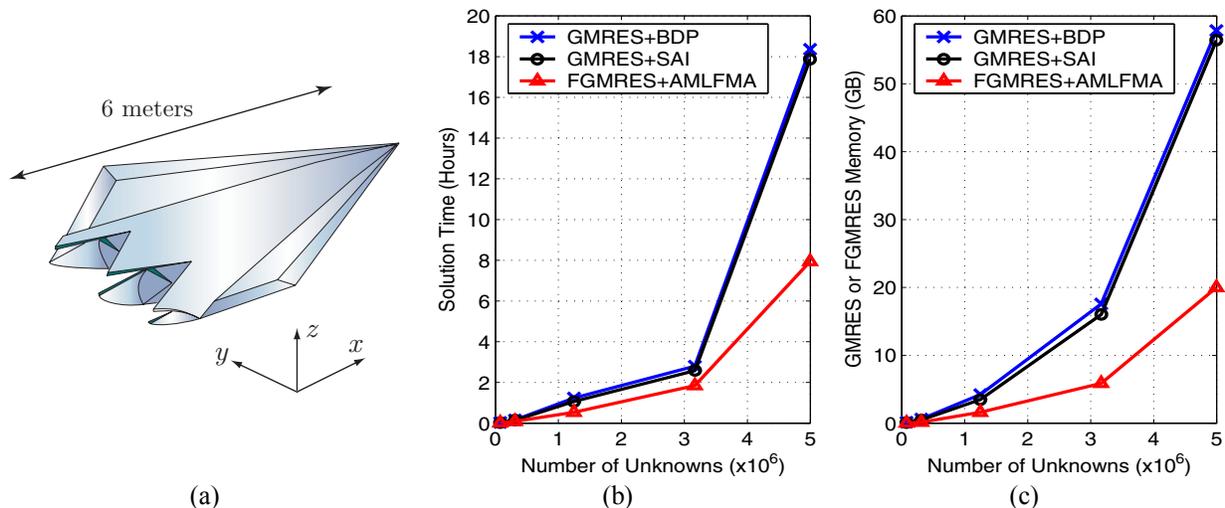


Figure 2. (a) Flamme, the stealth airborne target [4]. (b) Solution time and (c) memory required by GMRES and FGMRES methods with various preconditioning techniques for the solution of Flamme problems.

The AMLFMA preconditioner can significantly accelerate the iterative solutions of problems involving complicated targets. As an example, Fig. 2 presents the solution of scattering problems involving the Flamme, which is a stealth airborne target [4]. The largest dimension of the Flamme is 6 meters. The nose of the target is directed towards the x axis, as depicted in Fig. 2(a), and the target is illuminated by a plane wave propagating in the $-x$ direction. Figs. 2(b) and 2(c) depict the solution time and memory required by the iterative solver (GMRES or FGMRES), when the problem is solved in a range of frequencies from 1 GHz to 8 GHz. Discretizations of the geometry with $\lambda/10$ mesh size at these frequencies lead to between 80,000 and 5 million unknowns. In addition to the solutions with FGMRES accelerated with the AMLFMA preconditioner, we also present the solutions performed by GMRES using two common preconditioners, i.e., the block-diagonal preconditioner (BDP) and the sparse-approximate inverse (SAI) preconditioner. Both the BDP and the SAI preconditioner are constructed from the near-field interactions. Figs. 2(b) and 2(c) show that the processing time and memory increase proportionally to the number of unknowns, but the increase is slower using FGMRES and the AMLFMA preconditioner. For large problems, the AMLFMA preconditioner provides two-fold speedup compared to BDP and SAI. The memory requirement is also reduced significantly with the AMLFMA preconditioner; even though the memory required by FGMRES is typically twice that required by GMRES.

Using parallel MLFMA and robust preconditioning techniques, we are able to solve problems involving large and complicated targets. As an example, Fig. 3 presents the results of a scattering problem involving the Flamme geometry depicted in Fig. 2(a) at 16 GHz. At this frequency, the maximum dimension of the Flamme is 320λ . The target is illuminated by a plane wave propagating in the x - y plane at a 30° angle from the x axis, and the electric field is polarized in the z direction. Using $\lambda/10$ triangulation, the problem is discretized with 24,782,400 unknowns. The solution is performed by a 10-level MLFMA parallelized into 16 processes on a cluster of Intel Xeon processors. The time required for the setup and the iterative solution parts is 104 and 267 minutes, respectively. Fig. 3(a) presents the co-polar RCS values in dBm^2 on the x - y plane as a function of bistatic angle, where 30° and 210° correspond to backscattering and forward-scattering directions, respectively. In addition to the values obtained with MLFMA, we also present the RCS computed with a physical optics (PO) approximation. For a detailed comparison, backscattering and forward-scattering directions are magnified in Figs. 3(b) and 3(c), respectively. Fig. 3 shows that MLFMA and the PO approximation are consistent at some bistatic angles, such as the forward-scattering direction, in which the scattering is strong. However, the two results disagree at some other angles, especially where the scattering is weak, such as in the backscattering direction. For accurate investigations of complicated targets, such

as the Flamme, full-wave solutions obtained with parallel MLFMA are extremely important, because approximate methods (e.g., PO) may not provide accurate results, even when the objects are large.

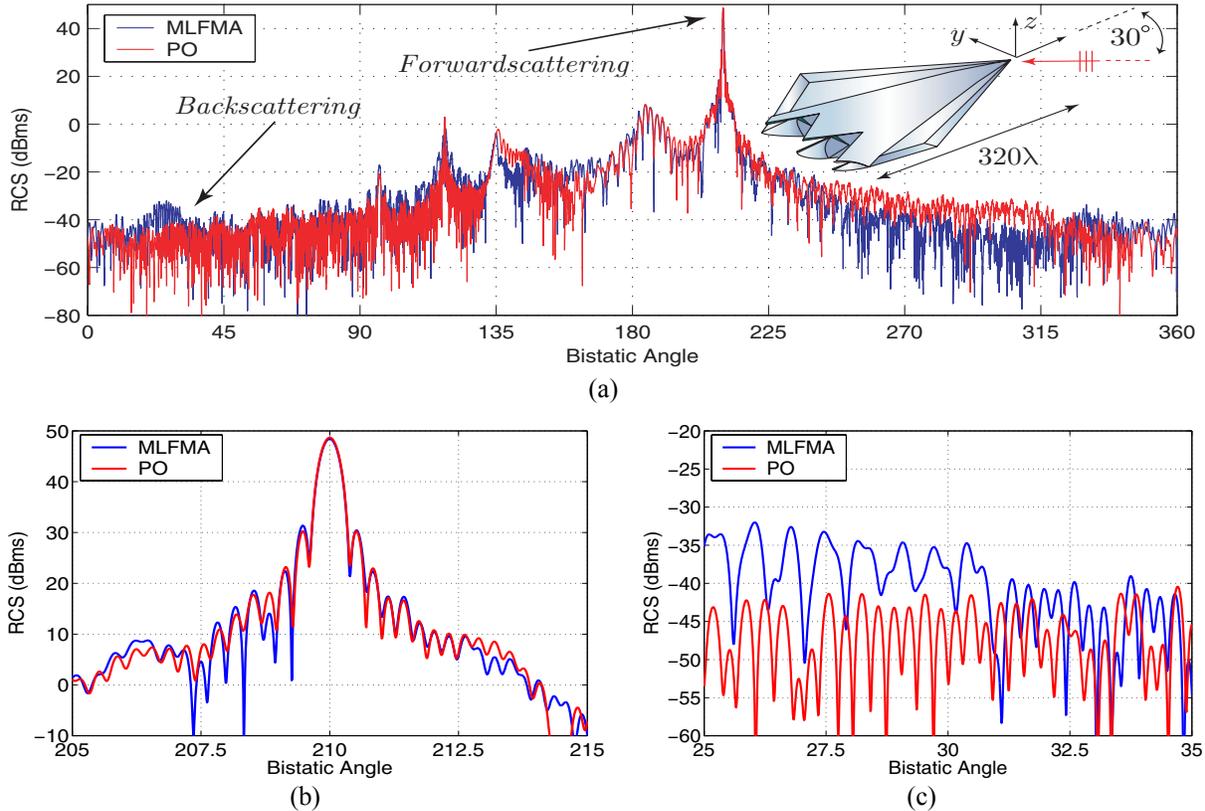


Figure 3. Bistatic RCS (in dBm^2) of the stealth airborne target Flamme at 16 GHz (a) from 0° to 180° , (b) from 205° to 215° , and (c) from 25° to 35° .

4. Acknowledgments

This work was supported by the Scientific and Technical Research Council of Turkey (TUBITAK) under Research Grants 105E172 and 107E136, by the Turkish Academy of Sciences in the framework of the Young Scientist Award Program (LG/TUBA-GEBIP/2002-1-12), and by contracts from **ASELSAN and SSM**. Computer time was provided in part by a generous allocation from Intel Corporation.

5. References

1. J. Song, C.-C. Lu, and W. C. Chew, "Multilevel Fast Multipole Algorithm for Electromagnetic Scattering by Large Complex Objects," *IEEE Trans. Antennas Propag.*, **45**, Oct. 1997, pp. 1488–1493.
2. Ö. Ergül and L. Gürel, "Hierarchical Parallelisation Strategy for Multilevel Fast Multipole Algorithm in Computational Electromagnetics," *Electronics Lett.*, **44**, Jan. 2008, pp. 3–5.
3. T. Malas, Ö. Ergül, and L. Gürel, "Approximate MLFMA as an Efficient Preconditioner," *International Symposium on Antennas and Propagation Proceedings*, Honolulu, Hawai'i, USA, 2007, pp. 1289–1292.
4. L. Gürel, H. Bağcı, J. C. Castelli, A. Cheraly, and F. Tardivel, "Validation Through Comparison: Measurement and Calculation of the Bistatic Radar Cross Section (BRCS) of a Stealth Target," *Radio Sci.*, **38**, Jun. 2003.