Fast-Multipole-Method Solution of Two-Dimensional Conductor Geometries

Levent Gürel
Dept. of Electrical and Electronics Engineering
Bilkent University
Bilkent, Ankara, Turkey

Electromagnetic radiation and scattering problems involving two-dimensional arbitrary conductor geometries are formulated using integral-equation techniques and solved using a number of methods including the fast multipole method (FMM). The excitation can be either an incident field or a finite source. Both TE and TM polarizations are considered. The current induced on the conductor geometry is discretized to convert the integral equation into a matrix equation. Equivalently, the discretized current elements can be thought of as two-dimensional subscatterers (strips) put together to construct the desired geometry.

In order to compare the accuracy and the efficiency of the FMM solution, two reference solutions are obtained. First, a direct solution technique (e.g., Gaussian elimination) is employed to solve the resulting matrix equation. Then, the same matrix equation is solved by using an iterative scheme (e.g., CG or GMRES), in which every iteration involved a matrix-vector multiplication. Finally, in a third solution, the ordinary matrix-vector multiplication is replaced by FMM in the same iterative scheme. The FMM employed here uses diagonalized translation operators for the two-dimensional Helmholtz equation. [V. Rokhlin, "Rapid solution of integral equations of scattering theory in two dimensions," *J. Comput. Phys.*, vol. 86, no. 2, pp. 414–439, Feb. 1990.]

It will be shown that the FMM requires $O(N^{1.5})$ operations per iteration as opposed to the matrix-vector multiplication being an $O(N^2)$ operation per iteration and the direct solution having $O(N^3)$ computational complexity. In addition to the estimates of computational complexities and memory requirements of these methods, their actual performances will also be compared and discussed. Examples of several two-dimensional arbitrary conductor geometries will be used in these comparisons.