EFIE and MFIE, Why the Difference?

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Abstract

EFIE (electric field integral equation) suffers from internal resonance, and the remedy is to
use MFIE (magnetic field integral equation) to come up with a CFIE (combined field inte-
gral equation) to remove the internal resonance problem. However, MFIE is fundamentally
a very different integral equation from EFIE. Many questions have been raised about the
differences.

First, it has often been observed that EFIE has better accuracy than MFIE. On the other
hand, MFIE has better convergence rate when solved with an iterative solver [1, 2]. Also,
EFIE has low-frequency breakdown, but MFIE does not have an apparent low-frequency
problem [3].

We will perform error analysis to explain why EFIE has better accuracy compared to MFIE
[4–7]. Mathematical analysis shows that EFIE has a smoothing operator, while MFIE has a
non-smoothing operator [8–10]. This difference often gives rise to better accuracy for EFIE
compared to MFIE.

MFIE is a second kind integral equation while EFIE is a first kind integral equation [10].
Hence, the eigenvalues of the EFIE operator tends to cluster around the origin, while the
eigenvalues of the MFIE operator are shifted away from the origin. Consequently, when
solved with an iterative solver, the convergence behavior of MFIE is superior to that of
EFIE.

It is well-known that EFIE suffers from the low-frequency breakdown problem. MFIE
does not suffer from apparent low-frequency breakdown, but it suffers from low-frequency
inaccuracy [3]. All these problems can be taken care of by performing the loop-tree decom-
position.

The EFIE operator is often known as the $L$ operator and the MFIE operator is often known
as the $K$ operator in the literature. The $L$ operator is a symmetric operator while the $K$
operator is an asymmetric operator. In some integral equations such as those involving
dielectric interfaces, these two operators appear simultaneously. They also appear concur-
rently in the invocation of the equivalence principle. Their discretization often gives rise
to an ill-conditioned matrix representation. We will discuss the reasons and present some remedies for them. More will be discussed at the conference presentation.

References


