EFIE and MFIE, Why the Difference?

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Abstract

EFIE (electric field integral equation) suffers from internal resonance, and the remedy is to use MFIE (magnetic field integral equation) to come up with a CFIE (combined field integral equation) to remove the internal resonance problem. However, MFIE is fundamentally a very different integral equation from EFIE. Many questions have been raised about the differences.

First, it has often been observed that EFIE has better accuracy than MFIE. On the other hand, MFIE has better convergence rate when solved with an iterative solver [1, 2]. Also, EFIE has low-frequency breakdown, but MFIE does not have an apparent low-frequency problem [3].

We will perform error analysis to explain why EFIE has better accuracy compared to MFIE [4–7]. Mathematical analysis shows that EFIE has a smoothing operator, while MFIE has a non-smoothing operator [8–10]. This difference often gives rise to better accuracy for EFIE compared to MFIE.

MFIE is a second kind integral equation while EFIE is a first kind integral equation [10]. Hence, the eigenvalues of the EFIE operator tends to cluster around the origin, while the eigenvalues of the MFIE operator are shifted away from the origin. Consequently, when solved with an iterative solver, the convergence behavior of MFIE is superior to that of EFIE.

It is well-known that EFIE suffers from the low-frequency breakdown problem. MFIE does not suffer from apparent low-frequency breakdown, but it suffers from low-frequency inaccuracy [3]. All these problems can be taken care of by performing the loop-tree decomposition.

The EFIE operator is often known as the $\mathcal{L}$ operator and the MFIE operator is often known as the $\mathcal{K}$ operator in the literature. The $\mathcal{L}$ operator is a symmetric operator while the $\mathcal{K}$ operator is an asymmetric operator. In some integral equations such as those involving dielectric interfaces, these two operators appear simultaneously. They also appear concurrently in the invocation of the equivalence principle. Their discretization often gives rise

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to an ill-conditioned matrix representation. We will discuss the reasons and present some remedies for them. More will be discussed at the conference presentation.

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