# Fast Algorithm for Electromagnetic Solution of Modified-Geometry Problems

*Levent Gürel*
*IBM Research Division*
*P.O. Box 218*
*Yorktown Heights, NY 10598*

## 1 Introduction

A fast algorithm is presented for the full-wave electromagnetic scattering solution of a class of modified-geometry problems. The algorithm assumes that the solution for a scattering structure, which, in general, may be composed of multiple scatterers, is known and stored. With the addition of a new scatterer to the problem (as in Figure 1), this algorithm avoids the solution of the modified problem from the beginning, but instead, makes use of the stored solution of the original structure. This algorithm, reduces not only the computation time, but also the computational complexity of the solution from $O(n^3)$ to $O(n^2)$, if there are $n$ unknowns in the problem.

## 2 Algorithm

A recursive implementation of the method of moments (MOM) for both spatial-domain and spectral-domain formulations was developed earlier [1] starting from the physical principles of electromagnetic radiation and scattering. Method of moments can be used to convert an operator equation $\mathcal{L}x = f$ to a matrix equation $\overline{\mathbf{L}} \cdot \mathbf{x} = \mathbf{f}$, which can be solved as $\mathbf{x} = \overline{\mathbf{L}}^{-1} \cdot \mathbf{f}$. If there are $N$ scatterers present in the problem, the matrices $\overline{\mathbf{L}}$ and $\overline{\mathbf{L}}^{-1}$ can be partitioned as

$$\overline{\mathbf{L}} = \begin{bmatrix} \overline{\mathbf{S}}_{11} & \overline{\mathbf{S}}_{12} & \cdots & \overline{\mathbf{S}}_{1N} \\ \overline{\mathbf{S}}_{21} & \overline{\mathbf{S}}_{22} & \cdots & \overline{\mathbf{S}}_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \overline{\mathbf{S}}_{N1} & \overline{\mathbf{S}}_{N2} & \cdots & \overline{\mathbf{S}}_{NN} \end{bmatrix} \qquad \overline{\mathbf{L}}^{-1} = \begin{bmatrix} \overline{\mathbf{I}}_{11(N)} & \overline{\mathbf{I}}_{12(N)} & \cdots & \overline{\mathbf{I}}_{1N(N)} \\ \overline{\mathbf{I}}_{21(N)} & \overline{\mathbf{I}}_{22(N)} & \cdots & \overline{\mathbf{I}}_{2N(N)} \\ \vdots & \vdots & \ddots & \vdots \\ \overline{\mathbf{I}}_{N1(N)} & \overline{\mathbf{I}}_{N2(N)} & \cdots & \overline{\mathbf{I}}_{NN(N)} \end{bmatrix} \tag{1}$$

Assuming that the $\overline{\mathbf{I}}_{ij(N-1)}$ matrices (for $i, j = 1, \ldots, N-1$), which represent the solution of the $(N-1)$-scatterer problem, are stored, the solution for the N-scatterer problem, i.e., the $\overline{\mathbf{I}}_{ij(N)}$ matrices (for $i, j = 1, \ldots, N$), can be computed using the following recursive relations [1]:

$$\overline{\mathbf{\Gamma}}_N = \overline{\mathbf{S}}_{NN} - \sum_{i=1}^{N-1} \sum_{j=1}^{N-1} \overline{\mathbf{S}}_{Ni} \cdot \overline{\mathbf{I}}_{ij(N-1)} \cdot \overline{\mathbf{S}}_{jN} \tag{2}$$

0-7803-1246-5/93/$3.00 

$$\bar{\mathbf{I}}_{NN(N)} = \bar{\mathbf{\Gamma}}_N^{-1} \tag{3}$$

$$\bar{\mathbf{I}}_{iN(N)} = -\left(\sum_{m=1}^{N-1} \bar{\mathbf{I}}_{im(N-1)} \cdot \bar{\mathbf{S}}_{mN}\right) \cdot \bar{\mathbf{\Gamma}}_N^{-1} \qquad i = 1, 2, \ldots, N-1 \tag{4}$$

$$\bar{\mathbf{I}}_{Nj(N)} = -\bar{\mathbf{\Gamma}}_N^{-1} \cdot \left(\sum_{m=1}^{N-1} \bar{\mathbf{S}}_{Nm} \cdot \bar{\mathbf{I}}_{mj(N-1)}\right) \qquad j = 1, 2, \ldots, N-1 \tag{5}$$

$$\bar{\mathbf{I}}_{ij(N)} = \bar{\mathbf{I}}_{ij(N-1)} + \left(\sum_{m=1}^{N-1} \bar{\mathbf{I}}_{im(N-1)} \cdot \bar{\mathbf{S}}_{mN}\right) \cdot \bar{\mathbf{\Gamma}}_N^{-1} \cdot \left(\sum_{n=1}^{N-1} \bar{\mathbf{S}}_{Nn} \cdot \bar{\mathbf{I}}_{nj(N-1)}\right) \qquad i, j = 1, 2, \ldots, N-1\,. \tag{6}$$

The above algorithm can be shown to be equivalent to general matrix inversion by partitioning [2] and, thus, can be applied in numerous disciplines of science. In the area of computational electromagnetics, the algorithm supplies the full-wave solution without having to make any approximations on the fundamental equations and boundary conditions, as opposed to, e.g, perturbation techniques.

## 3 Computational Complexity

Let the total number of unknowns defined for *all* of the $N-1$ scatterers be $n$ and the number of unknowns defined for the $N$th scatterer *alone* be $p$. Then, by counting the number of operations required by the algorithm given in Equations (2)–(6), it can be shown that the algorithm has $O(n^2p+p^3)$ computational complexity. Solving the $N$-scatterer problem using direct matrix inversion would require $O[(n+p)^3]$ operations. Clearly,

$$O(n^2p+p^3) < O[(n+p)^3] \tag{7}$$

for all positive integers $n$ and $p$. Furthermore,

$$O(n^2p+p^3) \longrightarrow O(n^2) \qquad \text{if } p \ll n. \tag{8}$$

This conclusion is also reached in [3] using the Sherman-Morrison-Woodbury formula.

## 4 Results

Figure 2 displays the actual computation times for electromagnetic scattering problems solved using the direct matrix inversion technique (solid curves) and the inversion-by-partioning technique (dashed curves). The computational times taken by the actual solution algorithm, matrix filling operations, and input/output (I/O) operations are displayed separately, in addition to their sum, the total computation time. The horizontal axis represents the number of scatterers, which may be arbitrary in general. In this study, the scatterers are chosen to be three-dimensional conducting patches. Each patch carries 24 current elements, i.e., 24 unknowns. Thus, the 100-scatterer problem represents a 2400-unknown problem. Each time a new scatterer is added to the problem, the complete problem is solved from the beginning using direct matrix inversion to obtain the solid curves. On the other hand, the dashed

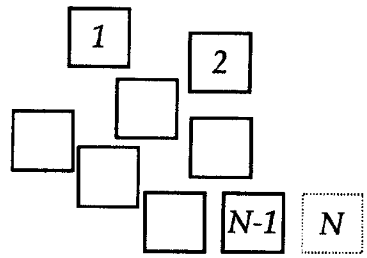


Figure 1: $N$th scatterer is introduced to the $(N-1)$-scatterer problem, whose solution is known and stored.

curves are obtained by reading in the stored solution for the original problem, using matrix inversion by partitioning to solve the modified problem, and writing out the solution for the modified problem for future use. The I/O time is compared to all of the solve, fill, and total computation times to demonstrate that it is not significant.

Since the horizontal and vertical axes of Figure 2 are scaled logarithmically, the slope of each curve is equal to the order of the computational complexity of the corresponding operation. For instance, one can see the $O(n^3)$ and the $O(n^2)$ variations of the solution times for the direct matrix inversion for the complete problem and the inversion by partitioning for the modified problem, respectively. Similarly, the matrix filling times for the complete problem and the modified problem vary as $O(n^2)$ and $O(n)$, respectively. The total computation times for both type of solutions are dominated by their corresponding matrix filling times. As the number of unknowns get larger, the solution times will dominate over the filling times, since the former have higher computational complexity than the latter. Furthermore, the matrix filling times can be reduced by taking advantage of the symmetries in the problem. However, these symmetries are deliberately not taken into consideration in this study in order to simulate the worst case.

# References

1. L. Gürel and W. C. Chew, "Recursive algorithms for calculating the scattering from $N$ strips or patches," *IEEE Trans. Antennas Propagat.*, vol. AP-38, pp. 507–515, April 1990.

2. C. Lanczos, *Applied Analysis*, p. 141, Prentice-Hall, Englewood Cliffs, N.J., 1956. (reprinted by Dover, New York, 1988.)

3. E. Yip and B. Tomas, "Obtaining scattering solutions for perturbed geometries and materials from moment method solutions," *Appl. Comput. Electromagn. Soc. J. and Newsletter*, vol. 3, no. 2, pp. 95–118, Fall 1988.

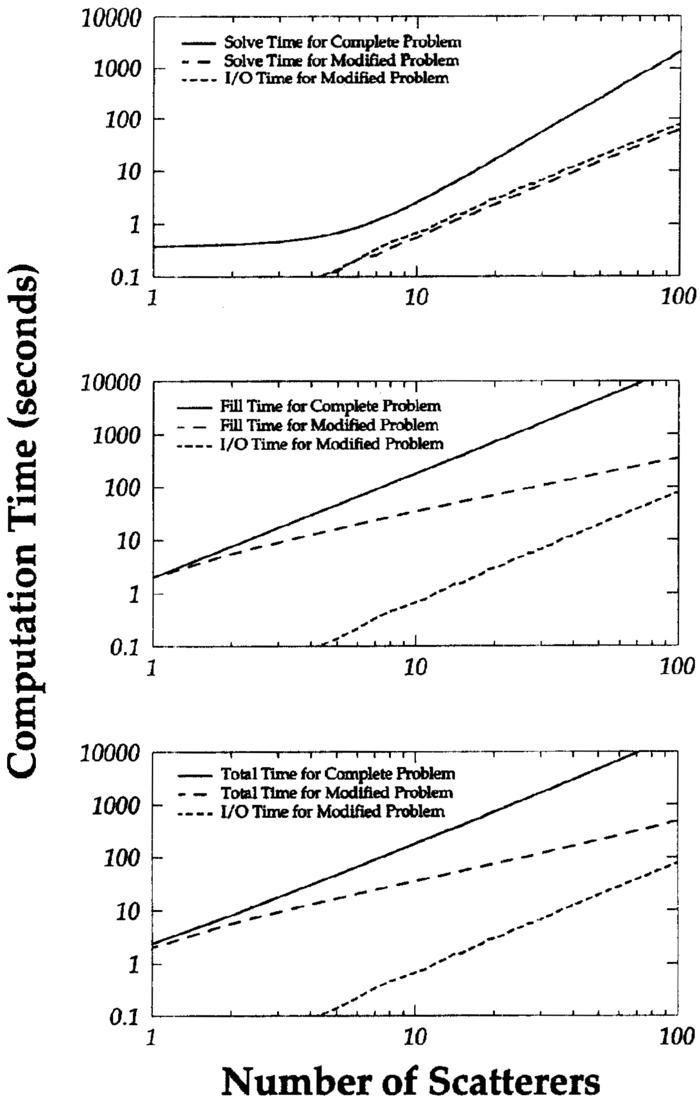


Figure 2: Matrix solution times, matrix filling times, I/O times, and the total computation times for the complete and modified problems solved using direct matrix inversion and inversion by partitioning, respectively.