

ACCURATE MODELING OF METAMATERIALS WITH MLFMA

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ABSTRACT

Electromagnetic modelling of large metamaterial (MM) structures employing multilevel fast multipole algorithm (MLFMA) is reported. MMs are usually constructed by periodically embedding unit cells, such as split-ring resonators (SRRs), into a host medium. Without utilizing any homogenization techniques, we accurately model large numbers of unit cells that translate into very large computational problems. By considering all of the electromagnetic interactions, the resulting dense matrix equations are solved iteratively with the accelerated matrix-vector products by MLFMA. To increase the efficiency, we also employ parallel computing in the solutions of large SRR problems.

Key words: Metamaterials; split-ring resonator; multi-level fast multipole algorithm.

1. INTRODUCTION

Since they were first proposed theoretically by Veselago in 1968 [1], metamaterials (MMs) have attracted a great amount of interest because of their unusual electromagnetic properties. MMs are usually constructed by periodically embedding unit cells, such as split-ring resonators (SRRs), into a host medium. In this paper, we specifically investigate the MM structures consisting of a single layer or multiple layers, each of which is an array of SRRs as depicted in Fig. 1(a). In our modelling and simulations, we take into account that these structures actually have finite extent and they exhibit interface properties. Without using homogenization, we accurately model large numbers of SRRs to understand the scattering and transmission properties of these structures. Accurate modelling and fast solution of these three-dimensional structures translate into very large computational problems, which can be solved with the aid of advanced acceleration techniques, such as the multilevel fast multipole algorithm (MLFMA) [2].

In this work, unit cell dimensions of the SRRs are in the order of microns to obtain an effective permeability around 100 GHz [3]. Around the resonance frequency, the transmission through the array is expected to decrease significantly due to the negative effective permeability stimulated in the medium. Dimensions of a single SRR depicted in Fig. 1(b) are as follows: The smaller ring has 43 μm inner radius and 67.2 μm outer radius, the larger ring has 80.7 μm inner radius and 107.5 μm outer radius, and the gap width is 7.2 μm . Single-layer SRR array in Fig. 1(a) is obtained by the arrangement of 11×18 SRRs. In this paper, we present the electromagnetic simulations of 1-layer and 2-layer SRR arrays.

2. INTEGRAL-EQUATION SOLUTIONS OF SRR STRUCTURES BY MLFMA

For the accurate simulations of MMs constructed by SRRs, we employ the electric-field integral equation (EFIE) to formulate the electromagnetic problem. For conducting surfaces, EFIE can be written in phasor notation with the $e^{-i\omega t}$ convention as

$$\hat{\mathbf{t}} \cdot \int_S d\mathbf{r}' \mathbf{J}(\mathbf{r}') \cdot \left(\bar{\mathbf{I}} - \frac{\nabla \nabla'}{k^2} \right) g(\mathbf{r}, \mathbf{r}') = \frac{i}{k\eta} \hat{\mathbf{t}} \cdot \mathbf{E}^i(\mathbf{r}) \quad (1)$$

directly from the boundary condition for the tangential electric field, where k and η are the wavenumber and wave impedance associated with the host medium. In our simulations, we assume that the relative permittivity of the host medium is 4.8, as it is commonly used in experimental setups [3]. In (1), scattered electric field is expressed in terms of the induced (unknown) surface current $\mathbf{J}(\mathbf{r})$ with the aid of the free-space Green's function

$$g(\mathbf{r}, \mathbf{r}') = \frac{e^{ikR}}{4\pi R} \quad \left(R = |\mathbf{r} - \mathbf{r}'| \right), \quad (2)$$

where \mathbf{r} is an observation point on the surface, and $\hat{\mathbf{t}}$ is the tangential vector at the observation point. On the right-hand side of (1), $\mathbf{E}^i(\mathbf{r})$ denotes the incident electric field.



Figure 1. (a) Single-layer SRR array obtained by the arrangement of 11×18 SRRs. (b) Induced surface current on a single SRR at 90 GHz.

As depicted in Fig. 1(b), SRRs are modelled by small triangles, on which Rao-Wilton-Glisson (RWG) [4] basis functions are defined to expand the unknown current density induced on the metallic surfaces as

$$\mathbf{J}(\mathbf{r}) = \sum_{n=1}^N a_n \mathbf{b}_n(\mathbf{r}). \quad (3)$$

In (3), a_n is the unknown coefficient of the n th basis function and N is the number of unknowns. We perform a convergence analysis for the optimal discretization in terms of the accuracy and efficiency, leading to 84 triangular elements for a single SRR. Projection of EFIE in (1) onto RWG testing functions \mathbf{t}_m , leads to the matrix equation

$$\sum_{n=1}^N Z_{mn}^E a_n = v_m^E, \quad m = 1, \dots, N, \quad (4)$$

where

$$Z_{mn}^E = \int_{S_m} d\mathbf{r} \mathbf{t}_m(\mathbf{r}) \cdot \int_{S_n} d\mathbf{r}' \overline{\mathbf{G}}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{b}_n(\mathbf{r}') \quad (5)$$

for

$$\overline{\mathbf{G}}(\mathbf{r}, \mathbf{r}') = \left(\overline{\mathbf{I}} - \frac{\nabla \nabla'}{k^2} \right) g(\mathbf{r}, \mathbf{r}') \quad (6)$$

represents the matrix element, and

$$v_m^E = \frac{i}{k\eta} \int_{S_m} d\mathbf{r} \mathbf{t}_m(\mathbf{r}) \cdot \mathbf{E}^i(\mathbf{r}) \quad (7)$$

represents the m th element of the excitation vector. In (5) and (7), S_m and S_n symbolize the spatial supports of the m th testing and n th basis functions, respectively. Solving for the coefficients in (4), we obtain accurate representations for the induced surface currents as depicted in Fig 1(b) for a single SRR.

The matrix equation in (4) is solved iteratively, where the matrix-vector products are accelerated by MLFMA. The fundamental idea in MLFMA is to replace the element-to-element interactions with cluster-to-cluster interactions in a multilevel scheme. This computational scheme relies on the factorization of the Green's function, which is valid only for basis and testing functions that are far from each other. For the far-field interactions, the matrix elements are derived as [5]

$$Z_{mn}^E = \frac{ik}{(4\pi)^2} \int d^2 \hat{\mathbf{k}} \mathbf{F}_{C_m}^{E,rec}(\hat{\mathbf{k}}) T_L(\mathbf{k}, \mathbf{D}) \cdot \mathbf{F}_{C'_n}^{E,rad}(\hat{\mathbf{k}}), \quad (8)$$

where $\hat{\mathbf{k}}$ is the angular direction on the unit sphere and

$$T_L(\mathbf{k}, \mathbf{D}) = \sum_{l=0}^L i^l (2l+1) h_l^{(1)}(kD) P_l(\hat{\mathbf{D}} \cdot \hat{\mathbf{k}}) \quad (9)$$

is the translation function written in terms of the spherical Hankel function of the first kind $h_l^{(1)}$ and Legendre polynomial P_l .

In (8), $\mathbf{F}_{C_m}^{E,rec}$ represents the receiving pattern of the m th testing function with respect to a close point C . Similarly, $\mathbf{F}_{C'_n}^{E,rad}$ is the radiation pattern of the n th basis function with respect to a close point C' . The translation function in (9) evaluates the interaction between the basis and testing groups that are located at C' and C , respectively, and separated by

$$\mathbf{D} = |\mathbf{D}| \hat{\mathbf{D}} = \mathbf{r}_C - \mathbf{r}_{C'}. \quad (10)$$

By employing MLFMA, matrix-vector multiplications can be performed in $O(N \log N)$ processing time using $O(N \log N)$ memory. In the iterative solutions of the scattering problems related to SRR structures, we

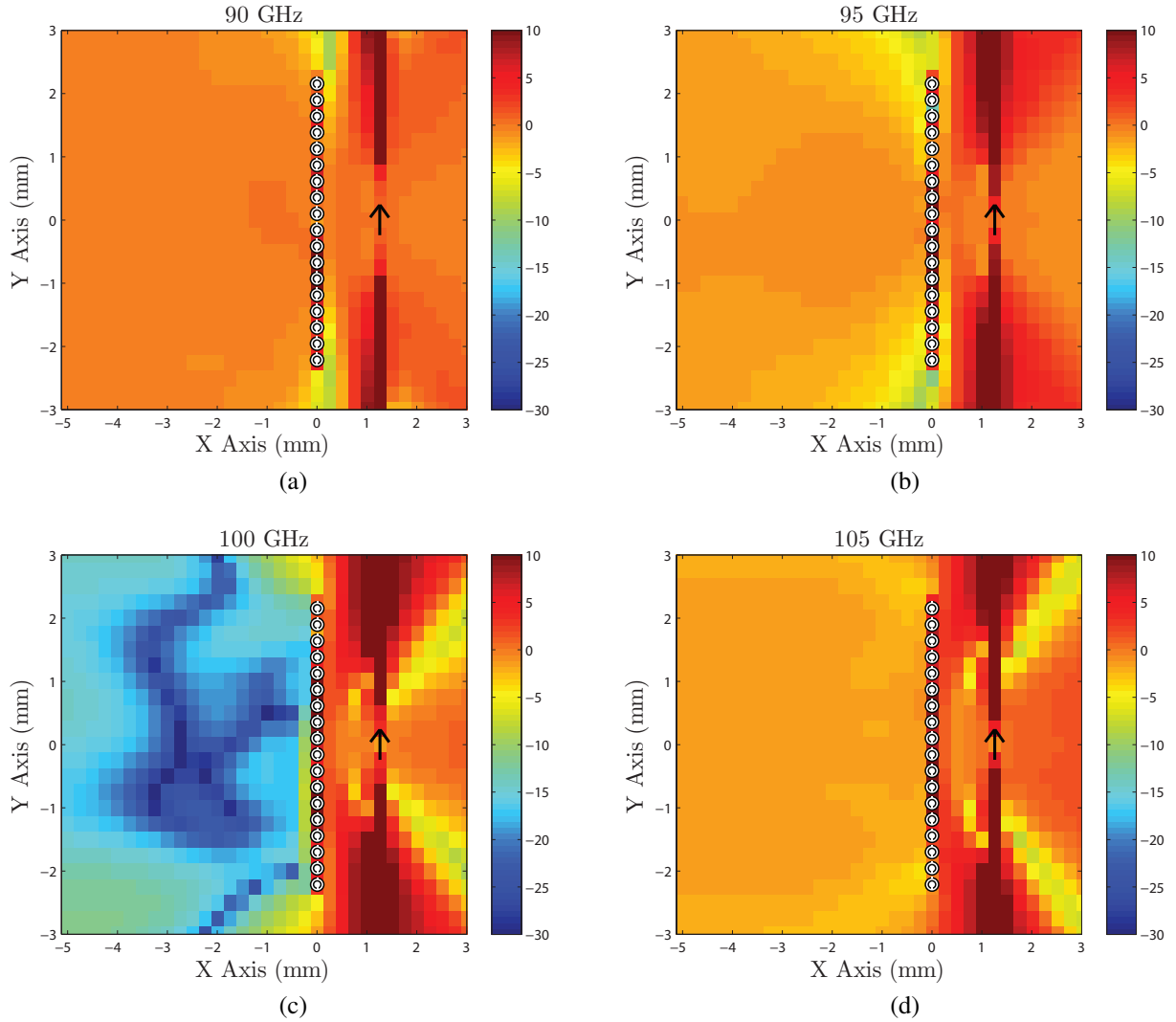


Figure 2. Power transmission for the 1-layer SRR array depicted in Fig. 1(a) at (a) 90 GHz, (b) 95 GHz, (c) 100 GHz, and (d) 105 GHz.

employ near-field preconditioner (NFP) to reduce the number of iterations. NFP is obtained by retaining all the near-field interactions of the problem. Although it is an expensive preconditioner, which has a higher complexity than MLFMA, the reduction in the processing time is significant for the solutions of SRR problems.

3. RESULTS

Fig. 2 presents the results for the 1-layer SRR array depicted in Fig. 1(a), where the power transmission is plotted at various frequencies. The transmission is calculated at different points in the $z = 0$ plane and the SRR array is depicted in the plots. The excitation is a Hertzian dipole oriented in the y direction as also indicated in the plots and the transmission region is on the left of the SRR array. At 90 GHz, 95 GHz, and 105 GHz, the power transmission through the SRR array is almost

unity, which corresponds to 0 decibels (dB). On the other hand, at the resonance frequency (100 GHz), the transmitted power drops dramatically due to the shadowing effect of the SRR array. In other words, the effective negative permeability introduced by the SRRs prevents the fields from passing into the transmission region.

When the number of layers is doubled and we obtain a $2 \times 18 \times 11$ array, the resulting power transmission is depicted in Fig. 3. In this case, we observe that the shadowing effect is not restricted to 100 GHz but instead extend from 95 GHz to 105 GHz. That is, the frequency region for the effective negative permeability gets wider for the 2-layer SRR array. We also select a location in the transmission region, which is the symmetric point of the location of Hertzian dipole with respect to the array, to present the power transmission as a function of frequency in Fig. 4(a). This way, we compare the 1-layer and 2-layer arrays and observe that the 2-layer array provides a wider band for low transmission.

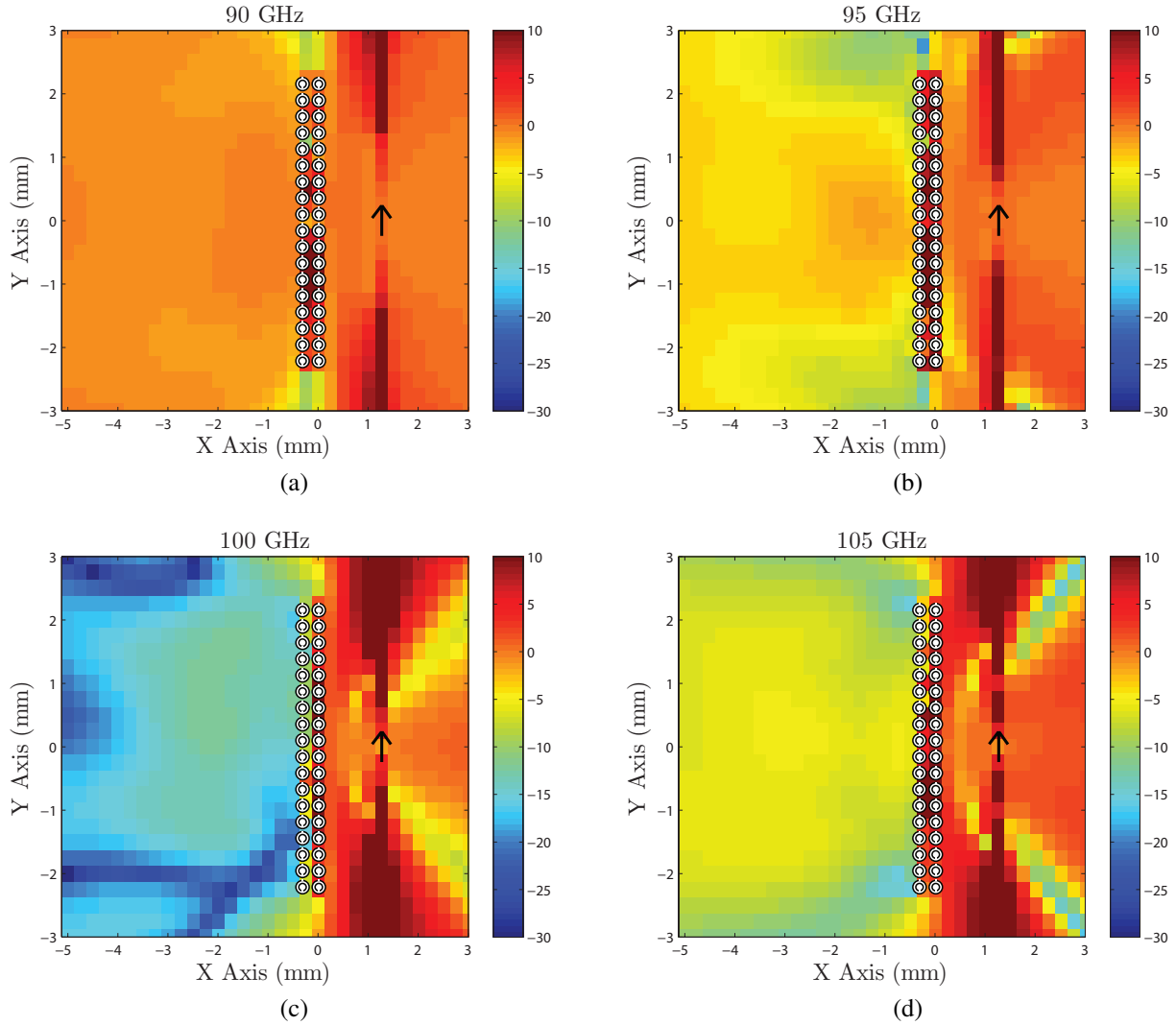


Figure 3. Power transmission for the $2 \times 18 \times 11$ SRR array at (a) 90 GHz, (b) 95 GHz, (c) 100 GHz, and (d) 105 GHz.

Finally, Fig. 4(b) presents the number of iterations for the solutions of 1-layer and 2-layer SRR arrays with respect to frequency. These iterations are obtained when NFP is employed, which includes all the available interactions of MLFMA. We observe that there exists a numerical resonance at 95 GHz and the number of iterations exceeds 2000. To obtain a convergence at these special frequencies, we reduce the number of levels of MLFMA and enrich the near-field matrix. For the SRR arrays with multiple layers, this strategy may fail to provide a solution with the existing computational resources. Then, we have to employ stronger and more efficient preconditioners based on advanced techniques, such as two-level preconditioning.

4. CONCLUSION

We report our efforts for accurate and efficient solutions of electromagnetic problems related to large MM structures constructed by SRRs. We formulate the problem by EFIE and solve the resulting dense matrix equations iteratively by MLFMA. In this paper, electromagnetic simulations of 1-layer and 2-layer SRR arrays are presented. Results for the larger number of layers will be included in the presentation.

ACKNOWLEDGMENTS

This work was supported by the Scientific and Technical Research Council of Turkey (TUBITAK) under Research Grant 105E172, by the Turkish Academy of Sciences in the framework of the Young Scientist Award Program (LG/TUBA-GEBIP/2002-1-12), and by contracts from ASELSAN and SSM.

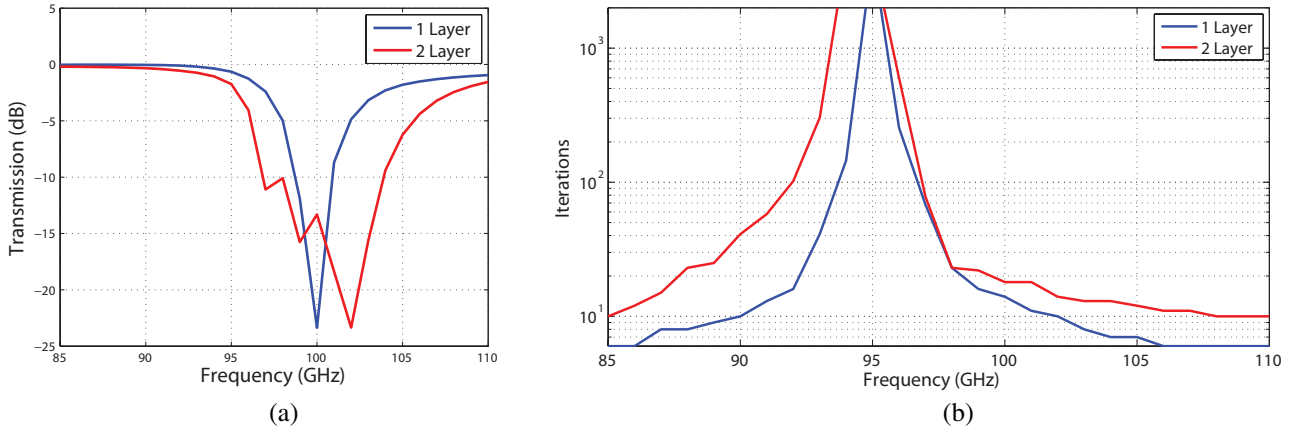


Figure 4. (a) Power transmission at a location in the transmission region, which is the symmetric point of the location of Hertzian dipole with respect to the array. (b) Iteration counts for the solutions of 1-layer and 2-layer SRR arrays.

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