A Windowed Recursive T-Matrix Algorithm for Wave-Scattering Solutions†

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I. Introduction

Recently, a recursive aggregate T-matrix algorithm (RATMA) has been developed using translation formulas [1,2]. The computational complexity of RATMA is $O(N^2)$ in two dimensions, and is $O(N^{7/3})$ in three dimensions, where N is the number of unknowns characterizing the scatterer [3]. RATMA has been used to calculate the volume scattering solution of E_z-polarized waves [4] showing reduced computational complexity. Moreover, the memory requirement of this algorithm is proportional to N rather than N^2 in a direct application of the method of moments.

When RATMA was applied to study the scattering of H_z-polarized waves by a two-dimensional inhomogeneous cylinder, it yielded unsatisfactory results. The the problem has been traced to the violation of the addition theorems. A generalized RATMA has been proposed to ameliorate this problem [5,6]. In this paper, we report another remedy so that RATMA works equally well for both the H_z- and E_z-polarized waves [3].

II. Restriction on the Addition Theorem

The addition theorem [2, p. 591],

$$H_m^{(1)}(k_0|\boldsymbol{\rho}-\boldsymbol{\rho}'|)e^{im\phi''} = \begin{cases} \sum_{n=-\infty}^{\infty} J_{n-m}(k_0\rho')H_n^{(1)}(k_0\rho)e^{in\phi-i(n-m)\phi'}, & \rho > \rho', \\ \sum_{n=-\infty}^{\infty} H_{n-m}^{(1)}(k_0\rho')J_n(k_0\rho)e^{in\phi-i(n-m)\rho'}, & \rho < \rho', \end{cases} \quad (1)$$

has its restricted regime of validity as indicated above In its valid regime, the addition theorem converges exponentially fast. However, when $\rho = \rho'$, the addition theorem may not converge. This stems from the fact that the right-hand side of (1) is a Fourier series expansion of its left-hand side for constant ρ's. Hence this series diverges when $\rho = \rho'$ because then $H_m^{(1)}(k_0|\boldsymbol{\rho}-\boldsymbol{\rho}'|)$ is a

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singular function of ϕ. However, the series can be made to converge on the $\rho = \rho'$ circle by using windowing techniques [3].

III. The Windowed Addition Theorem

In the practical implementation of (1), the series summation is truncated. For a Fourier series that is slowly convergent, this truncation of the spectrum in the Fourier space results in an oscillation in the real space. This oscillation, however, can be mitigated by windowing the Fourier spectrum. Consequently, a windowed version of the truncated summation in (2) would be

$$H_m^{(1)}(k_0|\rho-\rho'|)e^{im\phi''} = \begin{cases} \sum_{n=-L}^{L} J_{n-m}(k_0\rho')e^{-i(n-m)\phi'}W_n H_n^{(1)}(k_0\rho)e^{in\phi}, & \rho \geq \rho', \\ \sum_{n=-L}^{L} H_{n-m}^{(1)}(k_0\rho')e^{-i(n-m)\phi'}W_n J_n(k_0\rho)e^{in\phi}, & \rho \leq \rho', \end{cases} \tag{2}$$

where W_n is the weight for an appropriately chosen window. As an example, W_n is

$$W_n = \begin{cases} 1, & |n| \leq L - J, \\ \frac{1}{2}[1+\cos(\frac{|n|-L+J}{J}\pi)], & L-J < |n| < L, \\ 0, & |n| \geq L. \end{cases} \tag{3}$$

The above window has a flat section for $|n| \leq |L| - J$, and is tapered like the Hanning window for $|L| - J < |n| < L$. When $J = L$, it reduces to a complete Hanning window, and when $J = 0$, it is a box function.

Figure 1(a) shows the convergence of the addition theorem on the $\rho = \rho'$ circle with different values of J in (3). It is seen that increasing J generally increases the accuracy of the addition theorem on the $\rho = \rho'$ circle. In view of this, windowing is introduced into the addition theorems to improve on their convergence property on the $\rho = \rho'$ circle. This windowed addition theorem has been implemented in RATMA yielding a windowed RATMA (WRATMA). The windowing of the addition theorem does not alter the computational complexity of RATMA [3].

IV. Numerical Results

RATMA has been demonstrated to work for E_z-polarized waves [1-4], because for E_z-polarized waves, each subscatterer is scattering predominantly like a monopole. RATMA without windowing, however, does not work well for H_z-polarized waves. As will be shown, WRATMA gives good scattering solutions for H_z-polarized waves as well.

Figure 1(b) shows the scattering solution of an H_z-polarized incident wave on a circular dielectric cylinder which is about one wavelength in diameter. The WRATMA (squares) drastically improves the result over that for

RATMA (triangles). The CPU time for the windowed recursive algorithm on a CRAY-2 is about 6 s, but about 100 s for the method of moments.

Figure 1(c) shows the radar cross section (RCS) calculated with the windowed recursive algorithm for a three-wavelength circular dielectric scatterer. The method of moments becomes more costly for a scatterer of this size; hence, we compare our solution with the analytic solution. The agreement between the recursive algorithm and the analytic solution is good. The disagreement is due to imperfections in the physical modelling since the numerical model is not perfectly circular. The CPU time is about 78 s, and the number of subscatterers is 777.

Figure 1(d) shows the normalized RCS of an array of 81 strips. The strips here are not as tightly packed as those of the subscatterers of the inhomogeneous dielectric cylinder but the violation of the addition theorem is quite severe. Hovever, windowing mitigates the errors caused by this infraction. For the E_z-polarized wave, the windowed and unwindowed results are about the same. However, for the H_z-polarized wave, the windowed result is significantly better than the unwindowed result.

V. Conclusions

A windowed RATMA (WRATMA) can be used for both E_z and H_z wave scatterings. This algorithm has an $O(N^2)$ complexity in two dimensions and an $O(N^{7/3})$ complexity in three dimensions, which is faster than the method of moments with Gaussian elimination, and yet produces a solution valid for all angles of incidence.

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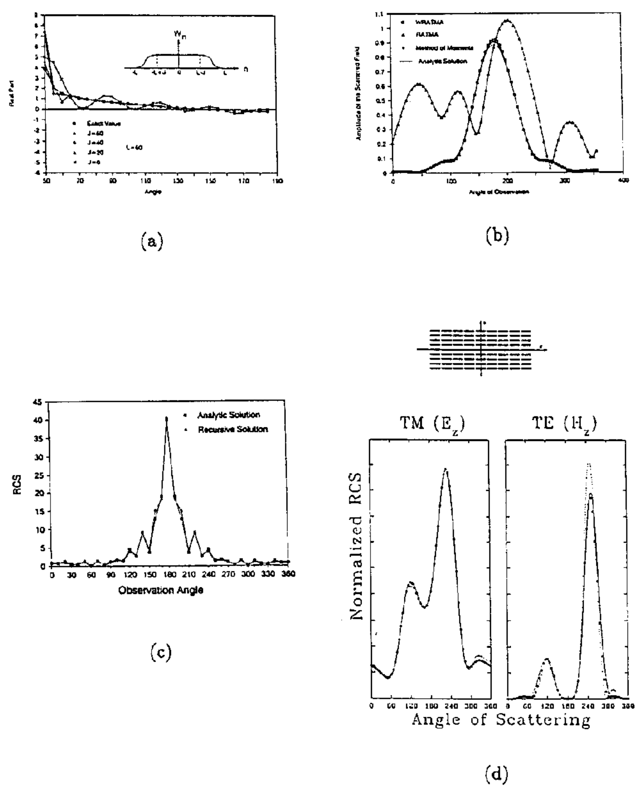

Figure 1.